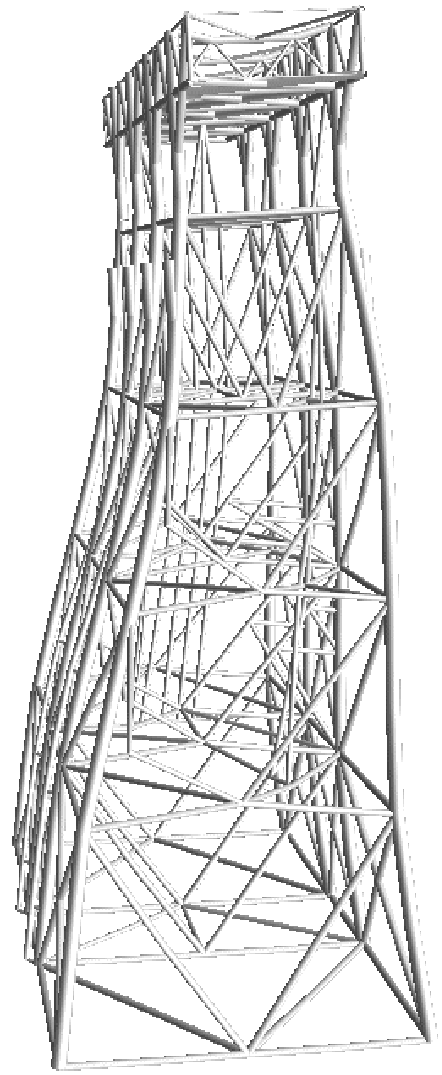
USFOS

Hydrodynamics

Theory
Description of use
Verification



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1. THEORY

1.1 Definitions and assumptions

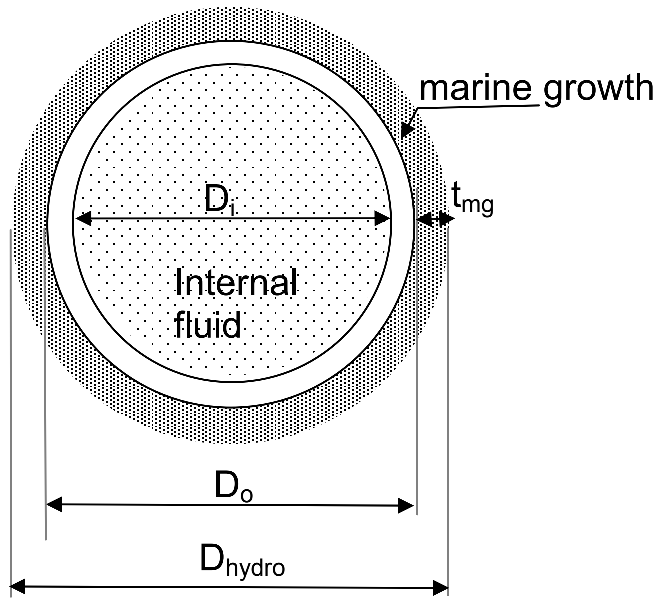


Figure 1.1 Pipe cross-sectional data

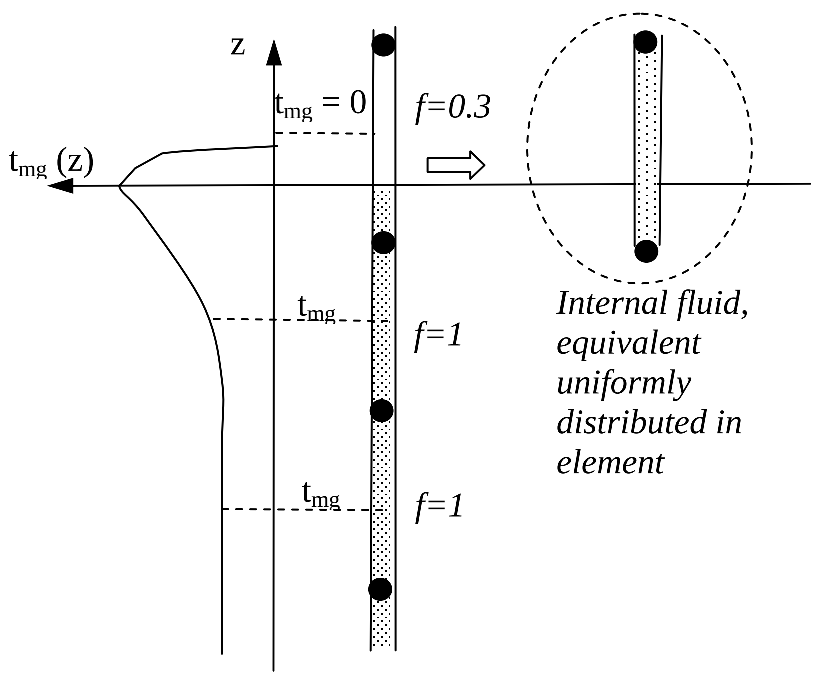


Figure 1.2 Marine growth and internal fluid definitions

D_o Outer diameter of tube
D_i Inner diameter of tube
ρ_s Steel density
ρ_{int} Density of internal fluid
ρ_w Density of sea water
f Fill ratio of internal fluid
C_M Added mass coefficient
C_D Drag coefficient
ρ_{mg} Average density of density of the marine growth layer including entrapped water
t_{mg} Thickness of marine growth
The thickness of marine growth is based on element *mid point* coordinate according to marine growth depth profile

Hydrodynamic diameter:

Net hydrodynamic diameter is assumed either equal to the tube diameter or as specified by input:

$$D_{hudro\_net} = \begin{matrix} D_o \\ D_{hydo\_net} \end{matrix}$$

Hydrodynamic diameter for wave force calculation, Morrison's equation:

$D_{hydro} = D_{hydro\_net} + 2t_{mg}$

$D_{drag} = D_{hydro}$ Diameter for drag force calculation

$D_{mass} = D_{hydro}$ Diameter for mass force calculation

Masses:

$\rho_s \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$ Mass intensity of tube

$\rho_{mg} \frac{\pi}{4}\left(\left(D_{hydro\_net} + 2t_{mg}\right)^2 - D_{hydro\_net}^2\right)$ Mass intensity of marine growth

$\rho_{\text{int}} \frac{\pi}{4} D_i^2 f$ Mass intensity of internal fluid, distributed uniformly over element length

$\rho_w \left(C_M - 1\right)\frac{\pi}{4} D_{hydro}^2$ Hydrodynamic added mass for dynamic analysis

Added mass intensity for each element is predefined. Motion in and out of water is taken into account on node level (consistent or lumped mass to nodes). Only submerged nodes contribute to system added mass.

Buoyancy forces:

$D_{buoyancy} = D_{hydro\_net}$ Buoyancy diameter (excluding marine growth)

$\rho_w g \frac{\pi}{4} D^2_{buoyancy}$ Buoyancy intensity of tube

$\rho_w g \frac{\pi}{4}\left(\left(D_{hydro\_net} + 2t_{mg}\right)^2 - D^2_{hydro\_net}\right)$ Buoyancy intensity of marine growth

Buoyancy forces are scaled according to BUOYHIST in dynamic analysis

Gravity forces

$\rho_s g \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$ Weight intensity of steel tube

$\rho_{mg} g \frac{\pi}{4}\left(\left(D_{hydro\_net} + 2t_{mg}\right)^2 - D^2_{hydro\_net}\right)$ Weight intensity of marine growth

$\rho_{\text{int}} g \frac{\pi}{4} D_i^2 f$ Weight intensity of internal fluid (distributed uniformly over element length)

Gravity forces are scaled according to relevant LOADHIST for gravity load case in dynamic analysis

1.2 Kinematics

1.2.1 Current

1.2.1.1 Introduction

Current is specified speed – depth profile and direction. The current speed is added vectorially to the wave particle speed for calculation of drag force according to Morrison’s equation.

1.2.1.2 Depth profile

A possible depth profile for current is illustrated in Figure 1.3 Values are given at grid points at various depths. The depth is specified according to a z- coordinate system, pointing upwards and with origin at mean sea surface level.

Tabulated values are taken from the table according to the element mid point. For intermediate depths values are interpolated. If member coordinate is outside the table values, the current speed factor is extrapolated.

Because wave elevation is taken into account the current speed factor should be given up to the maximum wave crest.

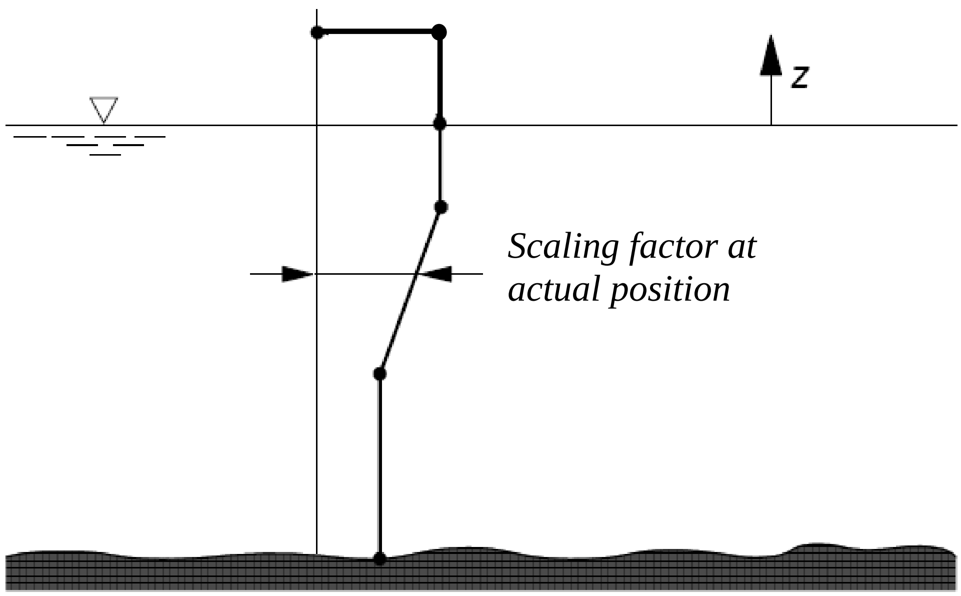


Figure 1.3 Depth profile for current speed factor coefficient

1.2.1.3 Direction Profile

Current is assumed to be uni-directional. The direction is specified in the same format as the wave direction.

1.2.1.4 Time dependency

Current speed may also be given a temporal variation. The entire depth profile is scaled this factor which is given as tabulated values as a function of time. Interpolation is used for time instants between tabulated points.

1.2.1.5 Current Blockage

1.2.2 Waves

1.2.2.1 Introduction

The following wave theories are available in USFOS:

- Linear (Airy) wave theory for infinite, finite and shallow water depth
- Stokes 5th order theory
- Dean's Stream function theory
- "Grid wave" – this option allows calculation of fluid flow kinematics by means of computational fluid dynamics (CFD). The forces from the kinematics may be calculated using USFOS force routines

Higher order wave theories are typically giving wave crests which are larger than wave troughs. This influences both the wave kinematics as such as well as actual submersion of members in the splash zone.

The most suitable wave theory is dependent upon wave height, the wave period and the water depth. The most applicable wave theory may be determined from the Figure 1.4 which is taken form API-RP2A (American Petroleum Institute, Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms)

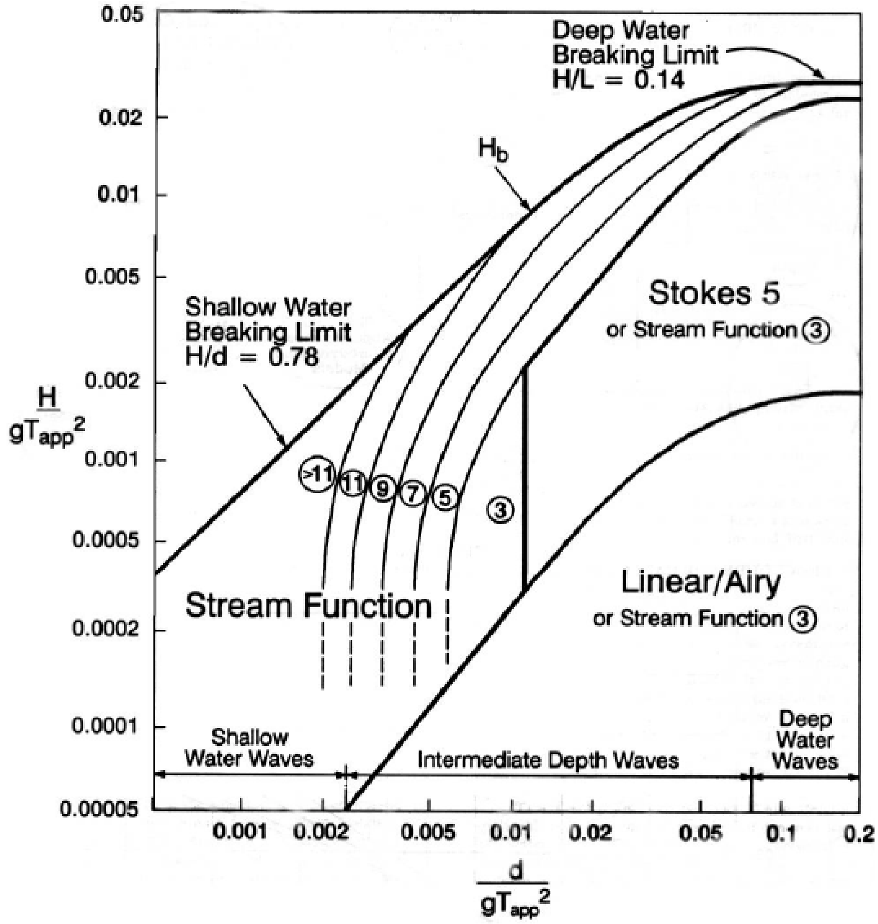


Figure 1.4 Applicability of wave theories

1.2.2.2 Airy

Airy waves are based on linear wave theory. Consider wave propagating along positive x-axis, as shown in Figure 1.5. The origin is located at sea surface with global z-axis pointing upwards.

The free surface level is given by

$$\eta = h\cos\left(\omega t - kx\right) \tag{1.1}$$

regardless of water depth.

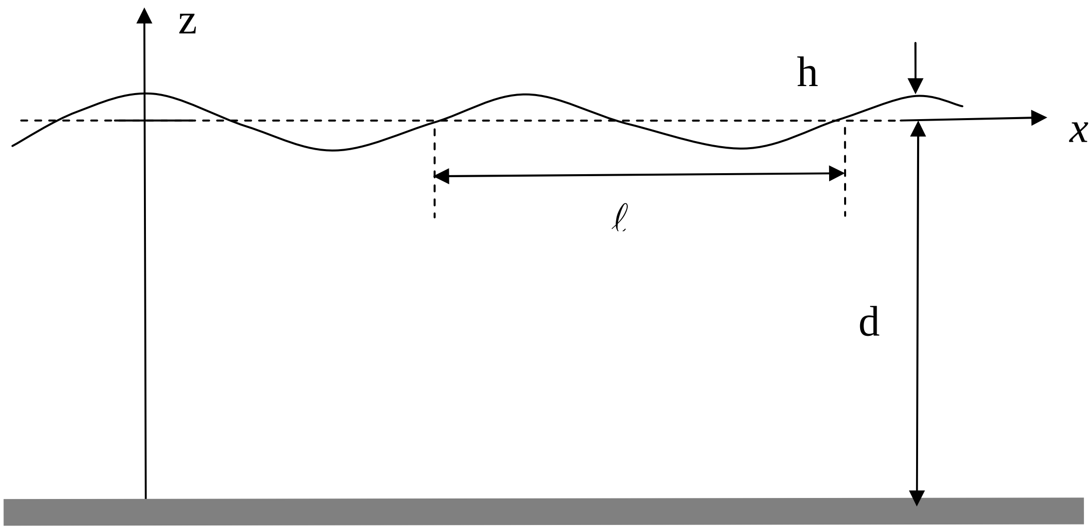


Figure 1.5 Wave definitions

Deep water waves: $\frac{d}{\lambda} > 0.5$

Deep water waves are assumed if the depth, d, is more than half of the wave length, i.e. the wave potential is given by

$$\phi = \frac{gh}{\omega} e^{-kz} \cos\left(\omega t - kx\right) \tag{1.2}$$

where

g = acceleration of gravity
h = wave amplitude
d = water depth
ω = circular wave frequency
k = wave number which is defined by:

$$\omega^2 = gk \tag{1.3}$$

For waves travelling at an angle θ with the x-axis the last sine term is replaced by

$$\cos\left(\omega t - k\cos\theta x - k\sin\theta y\right) \tag{1.4}$$

For simplicity formulas are expanded for waves travelling along positive x-axis.

The horizontal particle velocity (along x-axis) is given by

$$u = \frac{\partial \phi}{\partial x} = \omega h e^{kz} \sin\left(\omega t - kx\right) \tag{1.5}$$

and the vertical velocity

$$w = \frac{\partial \phi}{\partial z} = \omega h e^{kz} \cos\left(\omega t - kx\right) \tag{1.6}$$

Horizontal particle acceleration:

$$a_x = \frac{\partial u}{\partial t} = \omega^2 h e^{kz} \cos\left(kx - \omega t\right) \tag{1.7}$$

Vertical particle acceleration

$$a_z = \frac{\partial w}{\partial t} = -\omega^2 h e^{kz} \sin\left(kx - \omega t\right) \tag{1.8}$$

The hydrodynamic pressure is given by

$$p = -\rho g z + \rho g h e^{-kz} \sin\left(\omega t - kx\right) \tag{1.9}$$

where the first term is the static part and the 2nd term is the dynamic contribution.

Finite water depth: $0.05 < \frac{d}{\lambda} < 0.5$

The wave potential is given by

$$\phi = \frac{gh}{\omega} \frac{\cosh\left\{k\left(z + d\right)\right\}}{\cosh\left\{kd\right\}} \cos\left(\omega t - kx\right) \tag{1.10}$$

Where the wave number, *k*, is defined by:

$$\omega^2 = gk \tanh\left\{kd\right\} \tag{1.11}$$

The horizontal particle velocity (along x-axis) is given by

$$u = \frac{\partial \phi}{\partial x} = \omega h \frac{\cosh\left\{k\left(z + d\right)\right\}}{\sinh\left\{kd\right\}} \sin\left(\omega t - kx\right) \tag{1.12}$$

and the vertical velocity

$$w = \frac{\partial \phi}{\partial z} = \omega h \frac{\sinh\left\{k\left(z + d\right)\right\}}{\sinh\left\{kd\right\}} \cos\left(\omega t - kx\right) \tag{1.13}$$

Horizontal particle acceleration:

$$a_x = \frac{\partial u}{\partial t} = \omega^2 h \frac{\cosh\{k(z+d)\}}{\sinh\{kd\}} \cos(kx - \omega t) \tag{1.14}$$

Vertical particle acceleration

$$a_z = \frac{\partial w}{\partial t} = -\omega^2 h \frac{\sinh\{k(z+d)\}}{\sinh\{kd\}} \sin(kx - \omega t) \tag{1.15}$$

The hydrodynamic pressure is given by

$$p = -\rho g z + \rho g h \frac{\cosh\{k(z+d)\}}{\cosh\{kd\}} \sin(\omega t - kx) \tag{1.16}$$

where the first term is the static part and the 2nd term is the dynamic contribution.

Shallow water depth: $\frac{d}{\lambda} < 0.05$

The wave potential is given by

$$\phi = \frac{gh}{\omega} \cos(\omega t - kx) \tag{1.17}$$

Where the wave number, *k*, is defined by:

$$\omega^2 = gd \tag{1.18}$$

The horizontal particle velocity (along x-axis) is given by

$$u = \frac{\partial \phi}{\partial x} = \frac{\omega h}{gd} \sin(\omega t - kx) \tag{1.19}$$

and the vertical velocity

$$w = \frac{\partial \phi}{\partial z} = \omega h \left(1 + \frac{z}{d}\right) \cos(\omega t - kx) \tag{1.20}$$

Horizontal particle acceleration:

$$a_x = \frac{\partial u}{\partial t} = \frac{\omega^2 h}{kd} \cos(kx - \omega t) \tag{1.21}$$

Vertical particle acceleration

$$a_z = \frac{\partial w}{\partial t} = -\omega^2 h \left(1 + \frac{z}{d}\right) \sin(kx - \omega t) \tag{1.22}$$

The hydrodynamic pressure is expressed as

$$p = -\rho g z + \rho g h \sin\left(\omega t - kx\right) \tag{1.23}$$

where the first term is the static part and the 2nd term is the dynamic contribution.

Wave length

The wave length is given by:

$$\lambda = \frac{g}{2\pi} T^2 \tanh\left\{2\pi \frac{d}{\lambda}\right\} \tag{1.24}$$

It is observed that it is an implicit function of the wave length. For accurate solution iteration is required. Convergence may be slow close to shallow water.

Alternatively the following expressions are used:
Limiting period for deep water conditions, corresponding to wave length $\lambda = 2d$, where d is water depth: $T_{\lim} = \sqrt{\frac{2d}{g/2\pi}}$

If $T < T_{\lim}$ use deep water ($d/\lambda > 0.5$) : $\lambda = \frac{g}{2\pi} T^2$

If $T > T_{\lim}$ use finite water depth ((d/λ<0.5):'

$$\lambda = 2d + 2d\left(2\frac{T - T_{\lim}}{T_{\lim}}\right) = 2d\left(2\frac{T}{T_{\lim}} - 1\right)$$

The exact and the alternative formulation for wave length versus period are plotted in the Figure 1.6. It appears that the wave period is virtually linearly dependent on the wave period for finite water depth (versus the square of the period for infinite depth). The alternative formulation is very close to the exact formulation for the range of most practical applications. Some deviation is observed for very shallow water.

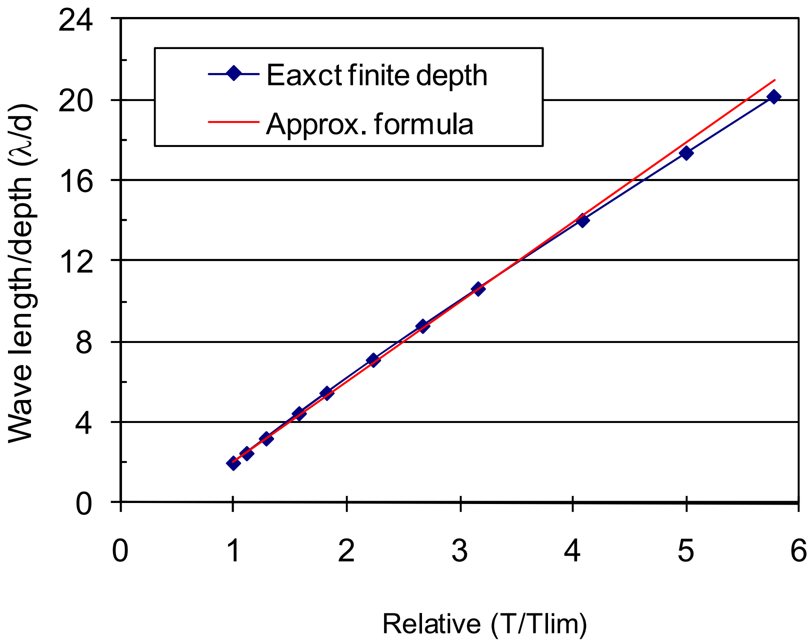


Figure 1.6 Wave length versus period for finite water depth

Extrapolated Airy theory

Airy wave theory is limited to infinitesimal waves. When eaves have finite amplitudes assumptions regarding wave kinematics must be introduced. One option is to use Airy wave kinematics up to surface elevation in wave troughs, excluding wave force calculation for members free from water. In wave crests, above mean sea level (z = 0), wave kinematics may be assumed constant equal to the value at z = 0. This procedure is called extrapolated Airy theory. The procedure is illustrated in Figure 1.7

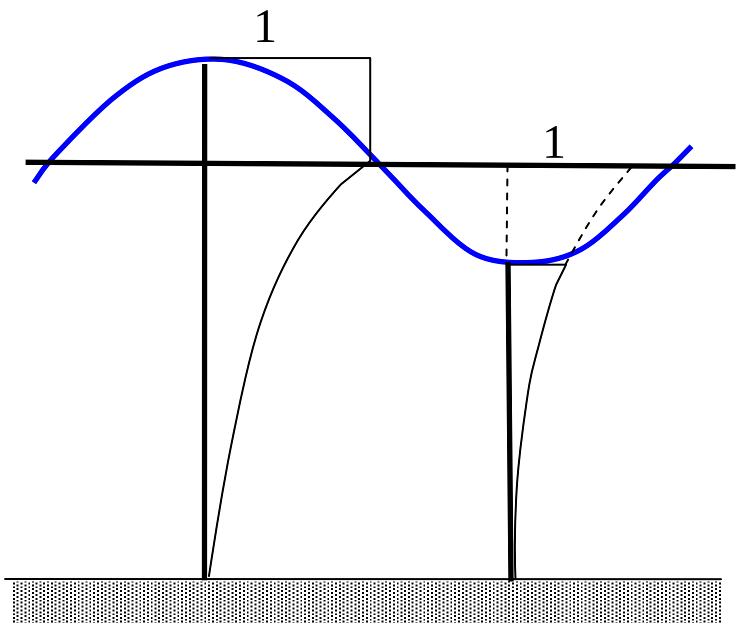


Figure 1.7 Illustration of extrapolated Airy theory

When the hydrodynamic pressure is integrated to true surface, the dynamic part is assumed constant above mean surface level in wave crests, while the "true" dynamic contribution is used below mean surface level in waves troughs. This is illustrated in Figure 1.8. It is observed that the total pressure vanishes exactly at wave crest surface, while a 2nd order error is introduced at wave trough surface.

The same approach is used for particle speed and accelerations, i.e. the values at mean sea surface, $z = 0$, is used for all $z > 0$.

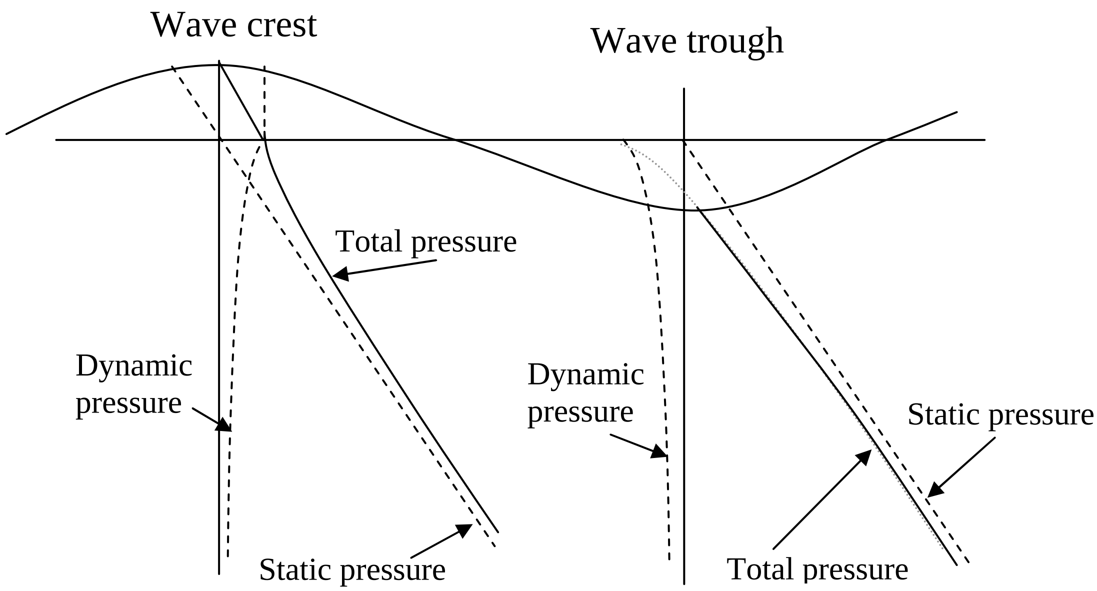


Figure 1.8 Hydrodynamic pressures in wave crests and wave troughs

It is observed that by this procedure there is no dynamic vertical force for bodies above z = 0 (only static buoyancy)

Stretched Airy theory (Wheeler modification)

By stretched Airy theory the kinematics calculated at the mean water level are applied to the true surface and the distribution down to the sea bed is stretched accordingly. This is achieved by substituting the vertical coordinate *z* with the scaled coordinate *z*' where

$$z' = (z - \eta)\frac{d}{d + \eta} \tag{1.25}$$

Where η is the instantaneous surface elevation

$$\eta = h\cos(kx - \omega t) \tag{1.26}$$

The procedure is illustrated in Figure 1.9.

It yields a dynamic vertical force for submerged bodies regardless of vertical location.

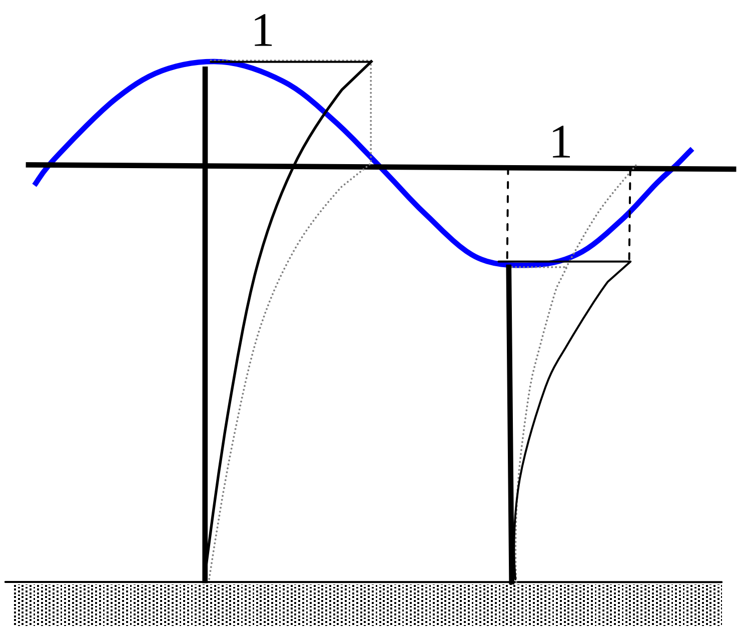


Figure 1.9 Illustration of stretched Airy (Wheeler) theory

Comparison of stretched – and extrapolated Airy theory

Stretched and extrapolated Airy theory yield different wave kinematics. The significance of the assumptions adopted with respect to Morrison force dominated structures depend on whether the mass force - or the drag force contribution predominates the wave loads. In Figure 1.10 and Figure 1.11 the forces histories for a mass dominated structure and an inertia dominated structure are plotted. The structure is a vertical column extending from sea floor at 85 m water depth and beyond wave crest. The diameter is 5.0 m in the mass dominated case and 0.5 m in the drag dominated case. The wave height and period are 18 m and 14 seconds, respectively.

The figure shows that the mass force calculated form the two theories differ little because the maximum force occurs when the acceleration term is maximum, i.e. with wave elevation approximately at mean water level (where the extrapolated and stretched Airy kinematics coincide). In this case the choice of wave kinematics assumption may not be important.

For drag dominated structure the difference in force level is substantial, because the maximum and minimum force levels occurs at wave crest and trough where the theories differ maximum.

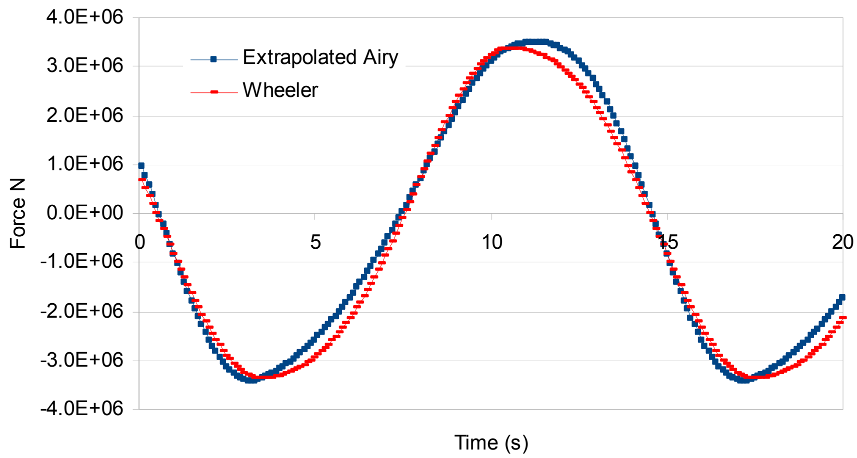


Figure 1.10 Comparison of stretched – and extrapolated Airy theory for mass force dominated column

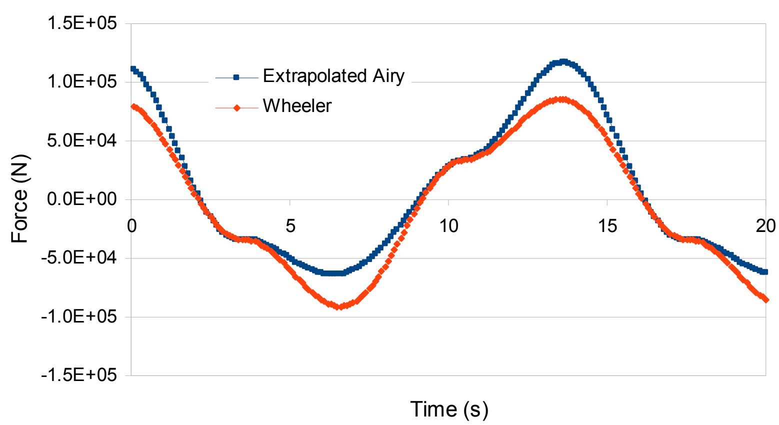


Figure 1.11 Comparison of stretched – and extrapolated Airy theory for drag force dominated column

1.2.2.3 Stoke's 5th order wave

The wave potential is given by a series expansion with five terms

$$\phi = \sum_{i=1}^{5} \phi_i^{'} \cosh\left\{k\left(z+d\right)\right\}\cos\left(\omega t - kx\right) \tag{1.27}$$

where

$$\begin{aligned} \phi_1^{'} &= \lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15} \\ \phi_2^{'} &= \lambda^2 A_{22} + \lambda^4 A_{24} \\ \phi_3^{'} &= \lambda^3 A_{33} + \lambda^5 A_{35} \\ \phi_4^{'} &= \lambda^4 A_{44} \\ \phi_5^{'} &= \lambda^5 A_{55} \end{aligned} \tag{1.28}$$

where

$$\begin{aligned} \eta_1^{'} &= \lambda \\ \eta_2^{'} &= \lambda^2 B_{22} + \lambda^4 B_{24} \\ \eta_3^{'} &= \lambda^3 B_{33} + \lambda^5 B_{35} \\ \eta_4^{'} &= \lambda^4 B_{44} \\ \eta_5^{'} &= \lambda^5 B_{55} \end{aligned} \tag{1.29}$$

The wave number, *k*, is defined by:

$$\omega^2 = gk \tanh\left\{kd\right\}\left(1 + \lambda^2 C_1 + \lambda^4 C_2\right) \tag{1.30}$$

The coefficients A,B and C are functions of kd only.

The horizontal particle velocity (along x-axis) is given by

$$u = \frac{\partial \phi}{\partial x} = \sum_{i=1}^{5} i \frac{\omega}{k} \phi_i^{'} \cosh\left\{k\left(z+d\right)\right\}\sin\left(\omega t - kx\right) \tag{1.31}$$

and the vertical velocity

$$w = \frac{\partial \phi}{\partial z} = \sum_{i=1}^{5} \frac{\omega}{k} \phi_i^{'} \sinh\left\{k\left(z+d\right)\right\}\cos\left(\omega t - kx\right) \tag{1.32}$$

Horizontal particle acceleration:

$$a_x = \frac{\partial u}{\partial t} = \sum_{i=1}^{5} i \frac{\omega^2}{k} \phi_i^{'} \cosh\left\{k\left(z+d\right)\right\}\cos\left(\omega t - kx\right) \tag{1.33}$$

Vertical particle acceleration

$$a_z = \frac{\partial w}{\partial t} = -\sum_{i=1}^{5} \frac{\omega^2}{k} \phi_i' \sinh\left\{k\left(z+d\right)\right\} \sin\left(\omega t - kx\right) \tag{1.34}$$

The hydrodynamic pressure is

$$p = -\rho g z - \rho \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2}\left[u^2 + w^2\right] \right\} \tag{1.35}$$

where the first term is the static part and the 2nd term is the dynamic contribution.

Starting with $L_5 = L_1$ and the parameter $\lambda_0 = 0$ the wave length is determined from the following iteration procedure

$$\lambda_i = \frac{\pi H}{L_5} - \lambda_{i-1}^3 B_{33} - \lambda_{i-1}^5 (B_{35} + B_{55}) \tag{1.36}$$

$$L_5 = L_0 \, Tanh(kd) \, (1 + \lambda_i^2 C_1 + \lambda_i^4 C_2), \quad k = \frac{2\pi}{L_5}$$

The wave celerity is given by:

$$c = \sqrt{(g \, Tanh(kd) \, (1 + \lambda^2 C_1 + \lambda^4 C_2) / \, k)}$$

The coefficients in Equation **(1.15)** are given by

$$B_{22} = \frac{\left(2Cosh(kd)^2 + 1\right) Cosh(kd)}{4Sinh(kd)^3}$$

$$B_{24} = Cosh(kd)(272Cosh(kd)^8 - 504Cosh(kd)^6 - 192Cosh(kd)^4 + 322Cosh(kd)^2 + 21) / 384Sinh(kd)^9$$

$$B_{33} = \frac{3(8Cosh(kd)^6 + 1)}{64 \, Sinh(kd)^6} \tag{1.37}$$

$$B_{35} = (88128 \, Cosh(kd)^{14} - 208224 \, Cosh(kd)^{12} + 70848 \, Cosh(kd)^{10} + 54000 \, Cosh(kd)^8 - 21816 \, Cosh(kd)^6 + 6264 \, Cosh(kd)^4 - 54 \, Cosh(kd)^2 - 81) / (12288 \, Sinh(kd)^{12} \, (6 \, Cosh(kd)^2 - 1))$$

$$B_{44} = Cosh(kd)(768Cosh(kd)^{10} - 448Cosh(kd)^8 - 48Cosh(kd)^6 + 48Cosh(kd)^4 + 106Cosh(kd)^2 - 21 / 384Sinh(kd)^9 \left(6Cosh(kd)^2 - 1\right)$$

$$B_{55} = (192000 \, Cosh(kd)^{16} - 262720 \, Cosh(kd)^{14} + 83680 \, Cosh(kd)^{12} + 20160 \, Cosh(kd)^{10} - 7280 \, Cosh(kd)^8 + 7160 \, Cosh(kd)^6 - 1800 \, Cosh(kd)^4 - 1050 \, Cosh(kd)^2 + 225) / (12288 \, Sinh(kd)^{10} \, (6 \, Cosh(kd)^2 - 1)(8 \, Cosh(kd)^4 - 11 \, Cosh(kd)^2 + 3))$$

$$C_1 = \frac{8\,Cosh(kd)^4 - 8Cosh(kd)^2 + 9}{8\,Sinh(kd)^4}$$

$$C_2 = (3840\,Cosh(kd)^{12} - 4096\,Cosh(kd)^{10} - 2592\,Cosh(kd)^8 - 1008\,Cosh(kd)^6 + 5944\,Cosh(kd)^4 - 1830\,Cosh(kd)^2 + 147) \,/\, (512\,Sinh(kd)^{10}\,(6\,Cosh(kd)^2 - 1)) \tag{1.38}$$

The wave elevation is given by

$$\eta = \sum_{i=1}^{5} E_i \,\frac{\sin\left\{i\left(\omega t - kx\right)\right\}}{k} \tag{1.39}$$

where the coefficients are given by the expressions

$$\begin{aligned} E_1 &= \lambda_1 \\ E_2 &= \lambda_1^{\,2} B_{22} + \lambda_1^4 B_{24} \\ E_3 &= \lambda_1^3 B_{33} + \lambda_1^5 B_{35} \\ E_4 &= \lambda_1^4 B_{44} \\ E_5 &= \lambda_1^5 B_{55} \end{aligned} \tag{1.40}$$

Horizontal velocity

$$u = \frac{\partial \phi}{\partial x} = \sum_{i=1}^{5} icD_i \cosh\left\{k\left(z+d\right)\right\}\sin\left\{i\left(\omega t - kx\right)\right\} \tag{1.41}$$

Vertical velocity

$$w = \frac{\partial \phi}{\partial z} = \sum_{i=1}^{5} icD_i \,\sinh\left\{ik\left(z+d\right)\right\}\cos\left\{i\left(\omega t - kx\right)\right\} \tag{1.42}$$

Horizontal particle acceleration:

$$a_x = \frac{\partial u}{\partial t} = \sum_{i=1}^{5} i\omega cD_i \,\cosh\left\{ik\left(z+d\right)\right\}\cos\left\{\grave{\imath}\left(\omega t - kx\right)\right\} \tag{1.43}$$

Vertical particle acceleration

$$a_z = \frac{\partial w}{\partial t} = -\sum_{i=1}^{5} i\omega cD_i \,\sinh\left\{ik\left(z+d\right)\right\}\sin\left\{\grave{\imath}\left(\omega t - kx\right)\right\} \tag{1.44}$$

The coefficients D_i are given by:

$$
\begin{aligned}
D_1 &= \lambda_1 A_{11} + \lambda_1^3 A_{13} + \lambda_1^5 A_{15} \\
D_2 &= \lambda_1^2 A_{22} + \lambda_1^4 A_{24} \\
D_3 &= \lambda_1^3 A_{33} + \lambda_1^5 A_{35} \\
D_4 &= \lambda_1^4 A_{44} \\
D_5 &= \lambda_1^5 A_{55}
\end{aligned}
\tag{1.45}
$$

$$
\begin{aligned}
A_{11} &= 1 / \text{Sinh}(kd) \\
A_{13} &= \frac{-\text{Cosh}(kd)^2 (5\,\text{Cosh}(kd)^2 + 1)}{8\,\text{Sinh}(kd)^5} \\
A_{15} &= -(1184\,\text{Cosh}(kd)^{10} - 1440\,\text{Cosh}(kd)^8 \\
&\quad - 1992\,\text{Cosh}(kd)^6 + 2641\,\text{Cosh}(kd)^4 - 249\,\text{Cosh}(kd)^2 + 18) \\
&\quad / (1536\,\text{Sinh}(kd)^{11}) \\
A_{22} &= \frac{3}{8\,\text{Sinh}(kd)^4} \\
A_{24} &= (192\,\text{Cosh}(kd)^8 - 424\,\text{Cosh}(kd)^6 - 312\,\text{Cosh}(kd)^4 + 480\,\text{Cosh}(kd)^2 - 17) \\
&\quad / (768\,\text{Sinh}(kd)^{10}) \\
A_{33} &= \frac{13 - 4\,\text{Cosh}(kd)^2}{64\,\text{Sinh}(kd)^7} \\
A_{35} &= (512\,\text{Cosh}(kd)^{12} + 4224\,\text{Cosh}(kd)^{10} - 6800\,\text{Cosh}(kd)^8 \\
&\quad - 12808\,\text{Cosh}(kd)^6 + 16704\,\text{Cosh}(kd)^4 - 3154\,\text{Cosh}(kd)^2 + 107) \\
&\quad / (4096\,(\text{Sinh}(kd)^{13})(6\,\text{Cosh}(kd)^2 - 1)) \\
A_{44} &= (80\,\text{Cosh}(kd)^6 - 816\,\text{Cosh}(kd)^4 + 1338\,\text{Cosh}(kd)^2 - 197) \\
&\quad / (1536\,(\text{Sinh}(kd)^{10})(6\,\text{Cosh}(kd)^2 - 1)) \\
A_{55} &= -(2880\,\text{Cosh}(kd)^{10} - 72480\,\text{Cosh}(kd)^8 + 324000\,\text{Cosh}(kd)^6 \\
&\quad - 432000\,\text{Cosh}(kd)^4 + 163470\,\text{Cosh}(kd)^2 - 16245) \\
&\quad / (61440\,(\text{Sinh}(kd)^{11})(6\,\text{Cosh}(kd)^2 - 1)(8\,\text{Cosh}(kd)^4 - 11\,\text{Cosh}(kd)^2 + 3))
\end{aligned}
\tag{1.46}
$$

1.2.2.4 Stream function

Stream function wave theory was developed by Dean (J. Geophys. Res., 1965) to examine fully nonlinear water waves numerically. It has a broader range of applicability than the Stokes' 5^{th} order theory. The method involves computing a series solution in sine and cosine terms to the fully nonlinear water wave problem, involving the Laplace equation with two nonlinear free surface boundary conditions (constant pressure, and a wave height constraint (Dalrymple, J.Geophys. Res., 1974)).

The order of the Stream function wave is a measure of how nonlinear the wave is. The closer the wave is to the breaking wave height, the more terms are required in order to give an accurate representation of the wave. In deep water, the order can be low, 3 to 5 say, while, in very shallow water, the order can be as great as 30. A measure of which order to use is to choose an order and then increase it by one and obtain another solution. If the results do not change significantly, then the right order has been obtained..

USFOS uses 10 terms as default.

For more information refer to Dean, R.G (1974, 972) and Dean and Dalrymple (1984)

When the wave height/depth is less than 0.5 the difference between stokes' 5^{th} order theory and Dean's theory is negligible.

1.2.2.5 Irregular Wave

In Fatigue Simulations (FLS) or Ultimate Limit State (ULS) analysis where dynamic effects, integration to true surface level, buoyancy effects, hydrodynamic damping and other nonlinear effects become significant, time domain simulation of irregular waves gives the best prediction of " reality".

For Irregular sea states wave kinematics may be generated on the basis of an appropriate wave spectrum. Two types of standard sea spectra are available; the Pierson-Moskowitz spectrum and the JONSWAP spectrum, refer Figure 1.12.

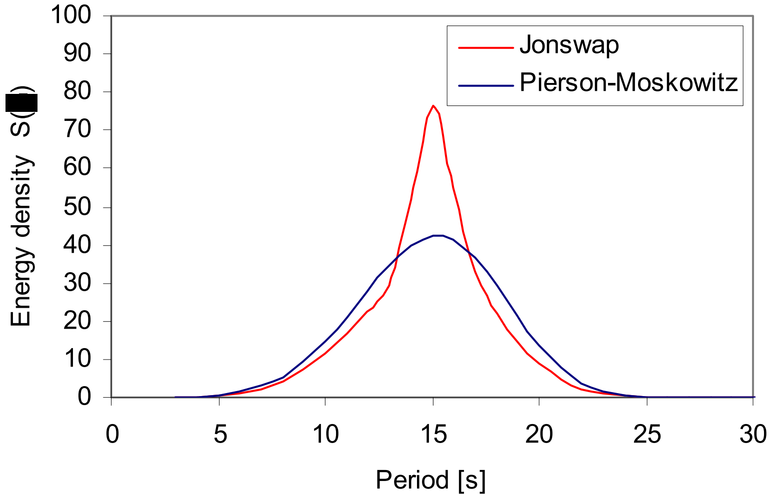


Figure 1.12 The JONSWAP and Pierson-Moskowitz wave spectra

The Pierson-Moskowitz spectrum applies to deep water conditions and fully developed seas. The spectrum may be described by the significant wave height, H_s, and zero up-crossing period, T_z:

$$S(\omega) = \frac{H_s^2 T_z}{8\pi^2}\left(\frac{\omega T_z}{2\pi}\right)^{-5} \exp\left[\frac{-1}{\pi}\left(\frac{\omega T_z}{2\pi}\right)^{-4}\right] \tag{1.47}$$

The JONSWAP spectrum applies to limited fetch areas and homogenous wind fields and is expressed bys. The spectrum may be described by the significant wave height, H_s, and zero up-crossing period , T_z:

$$S(\omega) = \alpha g^2 \omega^{-5} \exp\left[-1.25\left(\frac{\omega}{\omega_p}\right)^{-4}\right] \gamma^{\exp\left(-\left(\omega/\omega_p - 1\right)^2 / 2\sigma^2\right)} \tag{1.48}$$

where the peak frequency is given by

$$\omega_p = \frac{2\pi}{T_p} \tag{1.49}$$

and

$$\delta = 0.036 - \frac{0.0056 T_p}{\sqrt{H_s}} \tag{1.50}$$

$$\gamma = \exp\left[3.483\left(1 - \frac{0.1975\delta T_p^4}{H_s^2}\right)\right] \tag{1.51}$$

$$\alpha = 5.061(1-0.287\log\gamma)\frac{H_s^2}{T_p^4} \qquad (1.52)$$

$$\sigma = \begin{cases} \sigma_a = 0.07, & \omega \le \omega_p \\ \sigma_b = 0.09, & \omega > \omega_p \end{cases} \qquad (1.53)$$

T_p = Spectral peak period
γ = peak enhancement factor
σ_α = spectrum left width
σ_β = spectrum rigth width

The irregular sea elevation is generated by Fast Fourier Transform (FFT) of the wave energy spectrum. This gives a finite set of discrete wave components. Each component is expressed as a harmonic wave with given wave amplitude, angular frequency and random phase angle. By superposition of all extracted harmonic wave components with random phase angles uniformly distributed between 0 and 2π, the surface elevation of the irregular sea is approximated by

$$\eta(x,y,t) = \sum_{j=1}^{m} a_j \cos\left(\omega_j t - k_j \cos\theta x - k_j \sin\theta y - \phi_j\right)$$

Where
a_j = Amplitude of harmonic wave component j
ω_j = Angular frequency of harmonic component j
k_j = Wave number number for harmonic component j
ϕ_j = Random phase angle for harmonic component j

The procedure is illustrated in Figure 1.13

The amplitude of each harmonic component is determined by

$$a_j = \sqrt{2\int_{\omega_{l,j}}^{\omega_{u,j}} S(\omega)\,d\omega}$$

where $\omega_{l,j}$ and $\omega_{u,j}$ represent the lower and upper angular frequency limit for wave component j.

Two methods are available for the integration term.;

1) A constant angular frequency width is used, i.e. $\omega_{u,j} - \omega_{l,j} = \Delta\omega = \frac{\omega_u - \omega_l}{N}$ where ω_u and ω_λ are the upper and lower limit for integration of the wave energy spectrum.

2) The angular frequency limits are adjusted so that each component contains the same amount of energy, i.e

$a_j = \sqrt{2\int_{\omega_{l,j}}^{\omega_{u,j}} S(\omega)\,d\omega} = \sqrt{\frac{2\int_{\omega}^{\omega_u} S(\omega)\,d\omega}{N}}$ =constant This implies that all wave components have the same amplitude. The "density" of the wave components is larger in areas with much wave energy. The procedure is illustrated in Figure 1.14

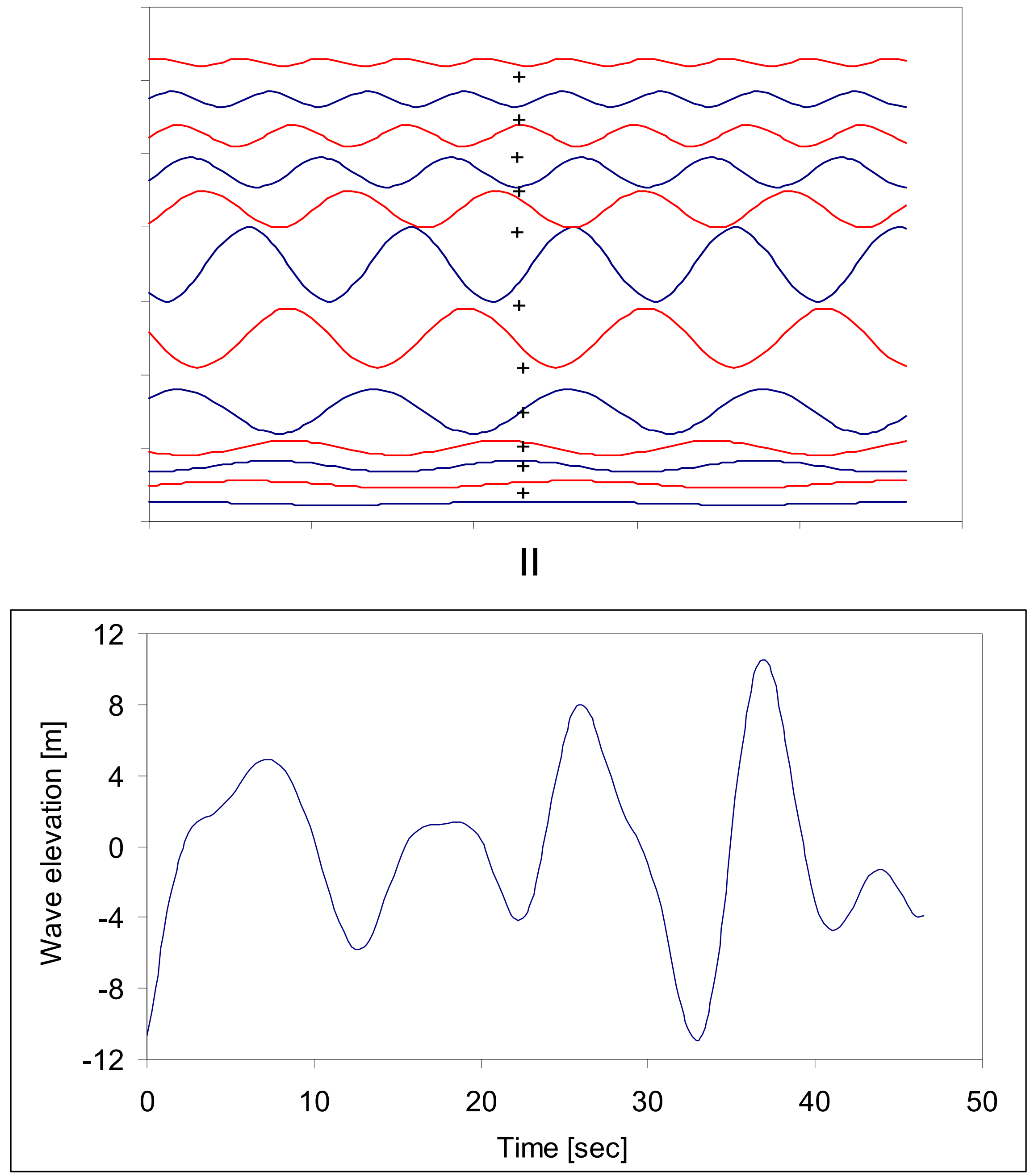


Figure 1.13 Illustration of irregular wave elevation history generated

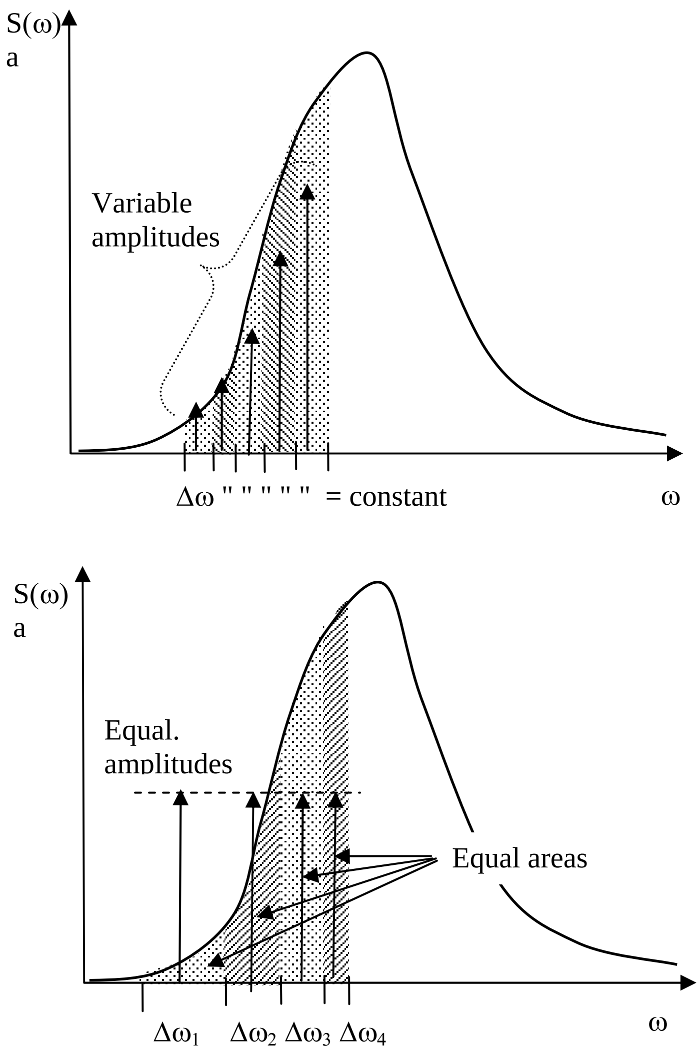


Figure 1.14 Illustration of irregular sea state generation

At each time instant loads are applied up to the instantaneous water surface, see Figure 1.13 generated by superposition of the regular wave components. On the basis of the kinematics of each wave components the hydrodynamic loads are calculated as a time series with a given time increment and for a given time interval.

In addition to surface waves the structure may also be exposed to a stationary current. The hydrodynamic forces are calculated according to Morison's equation with nonlinear drag formulation as described in Section 1.3 Buoyancy may calculated and added to the hydrodynamic forces for all members. The buoyancy may optionally be switched off for individual members.

The irregular sea simulating the specified sea state is generated by superposition of regular waves and thus linear wave theory is used. The irregular sea is generated by Fast Fourier Transform (FFT) of the wave energy spectrum. This gives a finite set of discrete wave components. Each component is expressed as a harmonic wave.

Irregular wave is defined similar to regular waves using the *WaveData* command. However, some additional parameters are required for an irregular wave specification:

- Hs and Tp (instead of Height and Period for a regular file)
- "Random Seed" parameter, (instead of phase)
- Wave energy spectrum (f ex JONSWAP or PM)
- Number of frequency components and the period range described by lower and upper period(T_{low}, T_{high}).
- Spectrum integration method (equal dω or constant area: $\int_{\omega}^{\omega_u} S(\omega)\,d\omega$

1.2.2.6 Grid Wave

---- To be completed ----

1.2.2.7 Riser Interference models

See www.USFOS.com under HYBER.

1.2.2.8 Initialization

In a dynamic simulation, the wave forces have to be introduced gradually, and the wave is ramped up using a user defined “envelope”. Discrete points as shown in Figure 1.15 define the envelope. The wave travels from left towards right, and the points are defined as X – scaling factor pairs. The wave height and thus the kinematics) are scaled. At time t=0, the wave will appear as shown in the figure, with “flat sea” to the right of point X_3, (where the structure is located). As time advances, the wave will move towards right, (in X-direction) and the wave height will increase. The ramp distance is typically 1 to 2 times the wavelength.

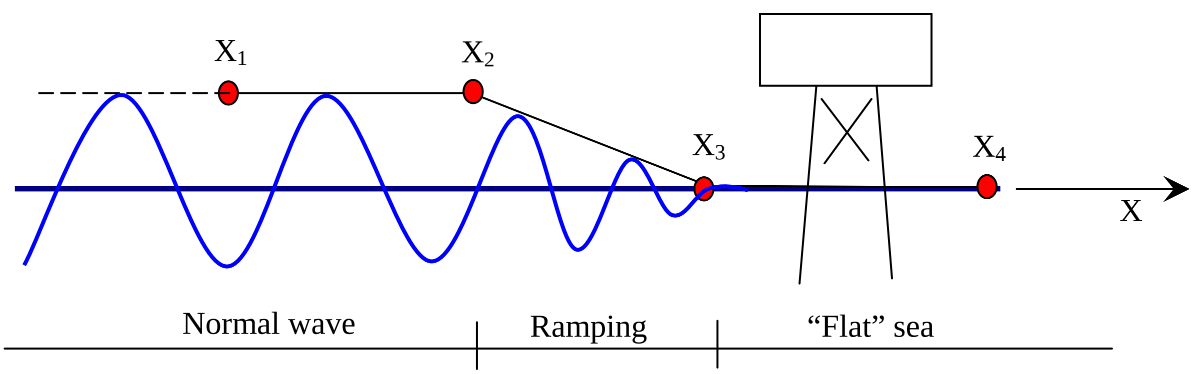


Figure 1.15 Initialization envelope

By default, the current acts with specified velocity from Time=0, but could be scaled (ramped) using the command `CURRHIST`.

1.2.2.9 "Spooling" of Irregular waves

A short term irregular sea is typically described for a duration of 3 hours. A time domain, nonlinear analysis of 3 hours may become overly demanding with respect to computational resources. Generally, only the largest extreme response is of interest, while the intermediate phases with moderate response is of little concern as regards ULS/ALS assessment. For this purpose the SPOOL WAVE command is useful. It will search for the n'th highest wave during the given storm and "spool" the wave up to a specified time before the actual peak. The analysis will then start the specified time before the peak, so that the structure is rapidly hit by the extreme wave without wasting simulation time.

Care should be exercised that the start up time is sufficiently long ahead of the peak wave, so that the initial transient response has been properly damped out. For very flexible structures (guyed towers etc.) where structural displacements may be relatively large the necessary start up period may be long.

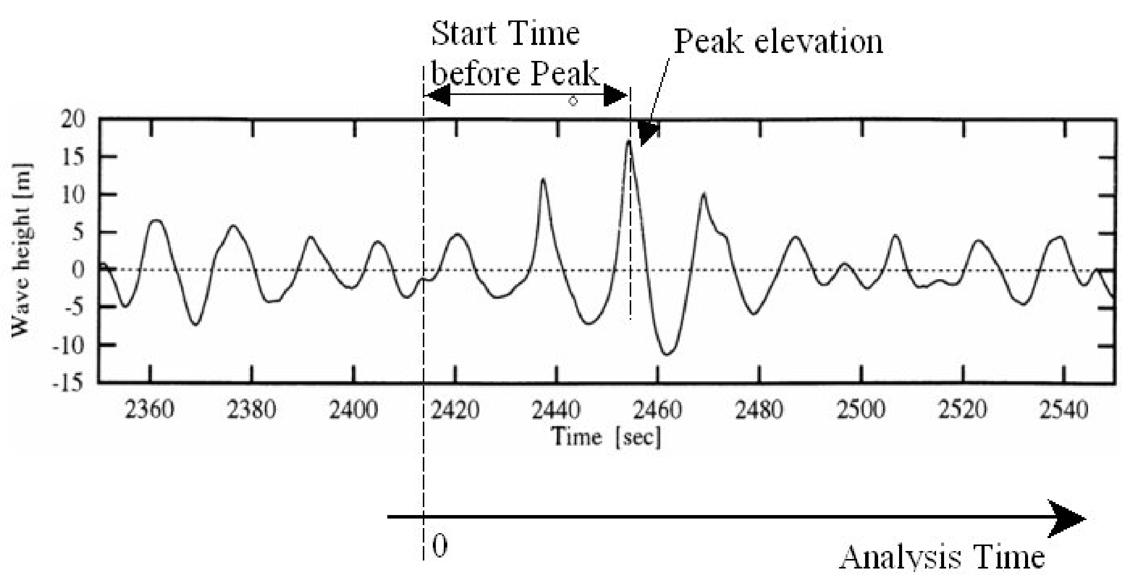


Figure 1.16 Principle sketch of SPOOL WAVE option: Skip simulation most of the time before the actual peak

1.2.2.10 Wave Kinematics Reduction

Wave kinematics given above is typically calculated for 2-dimesnional, i.e.long-crested, waves. Real waves are 3-dimensional, often characterised by a spreading function. 2-d theory may therefore overestimate true wave kinematics. A possible correction is to reduced 2-d particle velocities with kinematics reduction factor, KRF, such that

$$v_{corr} = v_{2D} \cdot KRF \tag{1.54}$$

This option is only implemented for Dean’s Stream theory

1.3 Force models

1.3.1 Morrison Equation

The wave force, dF, on a slender cylindrical element with diameter D and length ds is according to Morrison theory given by

$$dF = \left\{ \rho \frac{\pi}{4} D^2 C_M a_n \frac{1}{2} \rho C_D D v_n \left| v_n \right| \right\} ds \qquad (1.55)$$

where ρ is density of water, C_M is the mass coefficient and C_D is the drag coefficient, a_n is water particle acceleration and v_n is the water particle velocity including any current (wave velocity and current are added vectorially). The acceleration and velocity are evaluated *normal* to the pipe longitudinal axis. The drag term is quadratic. The sign term implies that the force changes direction when the velocity changes direction.

The total wave force is obtained by integrating eq. **(1.32)** along the member axis.

The component of the wave particle velocity normal to the tube longitudinal axis is evaluated as follows:

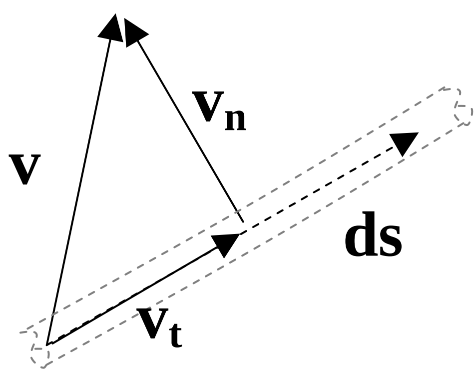


Figure 1.17 Vector representation of water particle velocity, v, and pipe segment, ds

With reference to Figure 1.17, let the pipe segment at the calculation point be represented by a unit vector along pipe axis

$$\mathbf{ds} = \frac{dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}}{ds}, \quad ds = \sqrt{dx^2 + dy^2 + dz^2} \qquad (1.56)$$

The wave particle velocity is represented by a vector

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k} \tag{1.57}$$

where v_x, v_y, and v_z represent the water particle velocity in x-, y- and z direction, respectively.

The component of the particle velocity along pipe axis is found from

$$\begin{aligned}\mathbf{v}_t &= |\mathbf{v}|\cos\alpha\,\mathbf{ds} = \frac{\mathbf{v}\bullet\mathbf{ds}}{ds^2}\left(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}\right) \\ &= \frac{v_x dx + v_y dy + v_z dz}{ds^2}\left(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}\right)\end{aligned} \tag{1.58}$$

where the sign • signifies the dot product of the two vectors. The normal velocity is accordingly given by

$$\mathbf{v}_n = \mathbf{v} - \mathbf{v}_t \tag{1.59}$$

with components

$$\begin{aligned}v_{x,n} &= v_x - \frac{v_x dx + v_y dy + v_z dz}{ds^2}dx \\ v_{y,n} &= v_y - \frac{v_x dx + v_y dy + v_z dz}{ds^2}dy \\ v_{z,n} &= v_z - \frac{v_x dx + v_y dy + v_z dz}{ds^2}dz \\ v_n &= \sqrt{v_{x,n}^2 + v_{y,n}^2 + v_{z,n}^2}\end{aligned} \tag{1.60}$$

In a similar manner the normal component of the water particle acceleration are calculated.

The three components of the Morrison wave force become

$$\mathbf{dF} = \begin{Bmatrix} df_x \\ df_y \\ df_z \end{Bmatrix} = \left\{ \rho\frac{\pi}{4}D^2C_M\begin{bmatrix} a_{x,n} \\ a_{y,n} \\ a_{z,n} \end{bmatrix}\frac{1}{2}\rho C_D D v_n \begin{vmatrix} v_{x,n} \\ v_{y,n} \\ v_{z,n} \end{vmatrix} \right\} ds \tag{1.61}$$

A slightly different procedure is actually adopted in USFOS. The water particle velocities and accelerations are first transformed to the element local axis system, refer Figure 1.18. As the local x-axis is oriented along the pipe axis, only the y- and z-component are of interest.

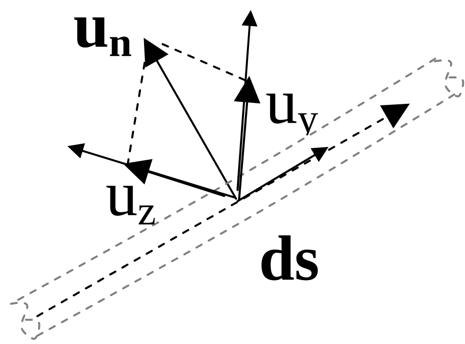


Figure 1.18 Water particle velocity in element local axis system

The Morrison wave force components in local axes are then given by

$$\mathbf{dF} = \begin{Bmatrix} df_y \\ df_z \end{Bmatrix} = \left\{ \rho \frac{\pi}{4} D^2 C_M \begin{bmatrix} \dot{u}_y \\ \dot{u}_z \end{bmatrix} \frac{1}{2} \rho C_D D u_n \begin{vmatrix} u_y \\ u_z \end{vmatrix} \right\} \mathbf{ds} \tag{1.62}$$

where

$$u_n = \sqrt{u_y^2 + u_z^2} \tag{1.63}$$

Subsequently, the forces are transferred to global system.

1.3.2 Influence of current

Current is characterized by a magnitude and direction and may be represented by a velocity vector. This vector is added vectorially to the water particle speed before transformation to element local axes.

1.3.3 Relative motion - drag force

If the structure exhibits significant displacement, the structure's own motion may start to influence the wave force. This influences the wave force and induces also hydrodynamic damping.

The effect may be included if the drag force is based upon the relative speed of the structure with respect to the wave.

The structure motion may be represented by a vector

$$\dot{\mathbf{x}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k} \tag{1.64}$$

Using the procedure given in Eqs **(1.57)-(1.60)** the velocity normal to the structure's axis, $\dot{\mathbf{x}}_n$, with components $\dot{x}_n, \dot{y}_n$ and $\dot{z}_n$, can be determined. The relative speed between the wave and the current is accordingly

$$\mathbf{v}_{rn} = \mathbf{v}_n - \dot{\mathbf{x}}_n \tag{1.65}$$

Hence, Eq. **(1.61)** may be used if $v_{x,n}$, $v_{y,n}$, $v_{z,n}$ v_n are substituted with the relative velocities given by:

$$\begin{aligned} v_{xr,n} &= v_{x,n} - \dot{x}_n \\ v_{yr,n} &= v_{y,n} - \dot{y}_n \\ v_{zr,n} &= v_{z,n} - \dot{z}_n \\ v_{rn} &= \sqrt{v_{xr,n}^2 + v_{yr,n}^2 + v_{zr,n}^2} \end{aligned} \tag{1.66}$$

Alternatively, the structure velocity is transformed to element local axes, before subtraction from the local wave – and current speed velocities. This is the approach adopted in USFOS

To account for relative velocity is optional in USFOS. When activated it is also possible to base the calculation of structure velocity on the average of the n preceding calculation steps.. Averaging may be introduced to soften the effect of high frequency vibrations. Default value is n = 0, i.e. no averaging is performed; only the last step is used.

1.3.4 Relative motion – mass force

The acceleration of the member, expressed as

$$\ddot{\mathbf{x}} = \ddot{x}i + \ddot{y}j + \ddot{z}k \tag{1.67}$$

influences the mass force in Morrison's equation.

The mass force depends upon the relative acceleration given by

$$\mathbf{dF}_m^{'} = \begin{Bmatrix} df_{mx} \\ df_{my} \\ df_{mz} \end{Bmatrix} = \rho \frac{\pi}{4} D^2 C_M \left\{ \begin{bmatrix} a_{x,n} \\ a_{y,n} \\ a_{z,n} \end{bmatrix} - \begin{bmatrix} \ddot{x}_n \\ \ddot{y}_n \\ \ddot{z}_n \end{bmatrix} \right\} ds \tag{1.68}$$

Part of this force is already taken into account in the dynamic equation system through the added mass term

$$diag\left[A_{11},A_{22},A_{33}\right]\begin{bmatrix}\ddot{x}_n\\ \ddot{y}_n\\ \ddot{z}_n\end{bmatrix}ds=\rho\frac{\pi}{4}D^2diag\left[\mathbf{1}\right]\begin{bmatrix}\ddot{x}_n\\ \ddot{y}_n\\ \ddot{z}_n\end{bmatrix}ds \tag{1.69}$$

This force must therefore be added on the right hand as well, giving the following net mass term:

$$\mathbf{dF}_m=\begin{Bmatrix}df_{mx}\\ df_{my}\\ df_{mz}\end{Bmatrix}=\rho\frac{\pi}{4}D^2\left\{C_M\begin{bmatrix}a_{x,n}\\ a_{y,n}\\ a_{z,n}\end{bmatrix}-\left[C_M-1\right]\begin{bmatrix}\ddot{x}_n\\ \ddot{y}_n\\ \ddot{z}_n\end{bmatrix}\right\}ds \tag{1.70}$$

Alternatively, the structure accelerations are transformed to element local axes, before subtraction from the local wave particle accelerations. This is the approach adopted in USFOS

1.3.5 Large volume structures

When the structure is large compared to the wave length Morrison’s theory is no longer valid. Normally this is assumed to be the case when the wave length/diameter ratio becomes smaller than five. For large diameter cylinders (relative to the wave length) Mac-Camy and Fuchs solution based on linear potential theory may be applied.

According to Mac-Camy and Fuchs theory the horizontal force, dF, per unit length, dz, of a cylinder in finite water depth is given by:

$$dF=\frac{4\rho gh}{k}\frac{\cosh\left\{k\left(z+d\right)\right\}}{\cosh\left\{kd\right\}}A\left(\pi\frac{D}{\lambda}\right)\cos\left(kx-\omega t+\alpha\right)dz \tag{1.71}$$

where A is a function of Bessel’s functions and their derivatives. The values of A and the phase angle α are tabulated in Table as function of the wave length/diameter ratio.

The Mac-Camy and Fuchs force corresponds to the mass term in the Morrison’s equation expressed as:.

$$dF=\omega^2h\frac{\pi}{4}D^2C_M\frac{\cosh\left\{k\left(z+d\right)\right\}}{\sinh\left\{kd\right\}}\cos\left(kx-\omega t+\alpha\right)dz \tag{1.72}$$

If the two expressions are put equal, the Mac-Camy and Fuchs force can be expressed in the Morrison mass term format. This yields the following equivalent mass coefficient:

$$C_M^{eq} = \frac{4}{\pi} \frac{A\left(\pi \frac{D}{\lambda}\right)}{\left(\pi \frac{D}{\lambda}\right)^2} \tanh\left(2\pi \frac{d}{\lambda}\right) \tag{1.73}$$

It is observed that the coefficient contains two contributions, which depend on:
- wave length/diameter ratio
- wave length/water depth ratio, for infinite water depth the factor is equal to unity

The exact values of equivalent mass coefficient can be calculated on the basis of the tabulated values of A. As shown in Figure 1.19 a good continuous fit to the tabulated values are obtained with the following function (infinite water depth used in plot):

$$C_M^{eq} = C_M \frac{1.05 \cdot \tanh\left(2\pi \frac{d}{\lambda}\right)}{\left\{abs\left(\pi \frac{D}{\lambda} - 0.2\right)^{2.2} + 1\right\}^{0.85}} \tag{1.74}$$

Noticing that the mass force is linear with respect to acceleration the modification of the mass term may alternatively be performed on the water particle acceleration to be used in the mass term calculation. The advantage with this method is that the modification may easily be carried out for each wave component in the irregular sea spectrum.

Hence the following modification is carried out on the acceleration term, while the mass coefficient is kept unchanged:

$$a^{eq} = a^{airy} \cdot \min\left[\frac{1.05 \cdot \tanh\left(2\pi \frac{d}{\lambda}\right)}{\left\{abs\left(\pi \frac{D}{\lambda} - 0.2\right)^{2.2} + 1\right\}^{0.85}},\quad 1\right] \tag{1.75}$$

An approximate function is also introduced for the phase angle α,

$$\alpha^{approx} = \frac{\pi}{180}\left\{-\frac{450}{8}\left(\pi \frac{D}{\lambda} - 2\right) - \frac{75}{\left(\pi \frac{D}{\lambda} + 0.5\right)^2}\right\} \text{ [radians]} \tag{1.76}$$

Figure 1.20 shows that the approximate expression for the phase angle is good in the range where the mass coefficient or acceleration term is modified, i.e. for wave length/diameter ratios smaller than 6.

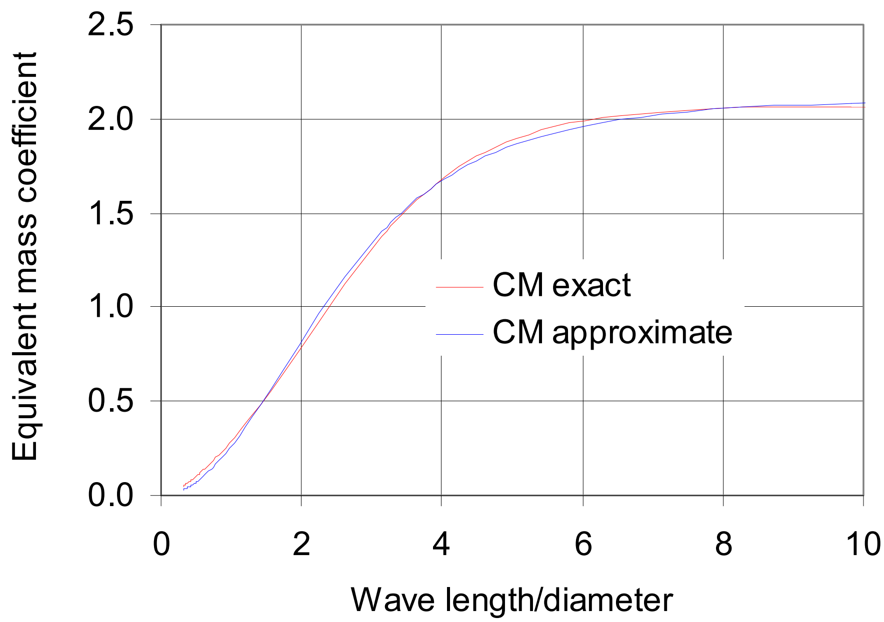


Figure 1.19 Exact and approximate equivalent mass coefficient for infinite water depth

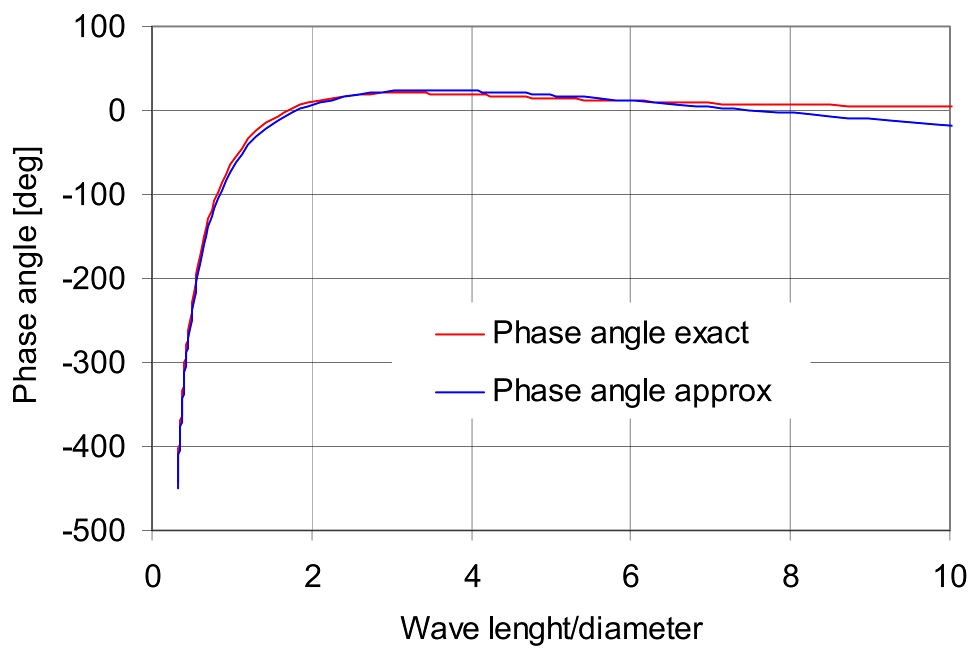


Figure 1.20 Exact and approximate phase angle in degrees

Table 1.1 Values of factor A, equivalent and approximate mass coefficient

wave length $\frac{\lambda}{D}$	$\pi \frac{D}{\lambda}$	$A\left(\pi \frac{D}{\lambda}\right)$	Phase $\alpha[\text{deg}]$	Exact C_M^{eq}	Approx. C_M^{eq}	Approx $\alpha[\text{deg}]$
157.0796	0.02	0.0006	0.02	1.91	2.06	-165.99
78.53982	0.04	0.0025	0.07	1.99	2.07	-146.95
52.35988	0.06	0.0057	0.16	2.02	2.08	-130.03
39.26991	0.08	0.0101	0.29	2.01	2.08	-114.95
31.41593	0.1	0.0159	0.45	2.02	2.09	-101.46
26.17994	0.12	0.0229	0.65	2.02	2.09	-89.36
22.43995	0.14	0.0313	0.89	2.03	2.10	-78.48
19.63495	0.16	0.0409	1.16	2.03	2.10	-68.68
17.45329	0.18	0.052	1.47	2.04	2.10	-59.82
15.70796	0.2	0.0643	1.82	2.05	2.10	-51.81
14.27997	0.22	0.078	2.20	2.05	2.10	-44.55
13.08997	0.24	0.093	2.61	2.06	2.10	-37.96
12.08305	0.26	0.1094	3.06	2.06	2.10	-31.97
11.21997	0.28	0.127	3.54	2.06	2.09	-26.52
10.47198	0.3	0.1459	4.05	2.06	2.09	-21.56
9.817477	0.32	0.1661	4.59	2.07	2.08	-17.04
9.239978	0.34	0.1874	5.15	2.06	2.08	-12.92
8.726646	0.36	0.2099	5.74	2.06	2.07	-9.16
8.267349	0.38	0.2335	6.35	2.06	2.06	-5.72
7.853982	0.4	0.2581	6.98	2.05	2.05	-2.59
7.479983	0.42	0.2836	7.63	2.05	2.04	0.26
7.139983	0.44	0.3101	8.29	2.04	2.03	2.87
6.829549	0.46	0.3373	8.96	2.03	2.01	5.24
6.544985	0.48	0.3653	9.64	2.02	2.00	7.41
6.283185	0.5	0.3938	10.32	2.01	1.98	9.38
6.041524	0.52	0.4229	11.00	1.99	1.96	11.16
5.817764	0.54	0.4523	11.67	1.97	1.95	12.78
5.609987	0.56	0.4821	12.34	1.96	1.93	14.25
5.416539	0.58	0.5122	13.00	1.94	1.91	15.57
5.235988	0.6	0.5423	13.64	1.92	1.89	16.77
5.067085	0.62	0.5725	14.27	1.90	1.87	17.84
4.908739	0.64	0.6025	14.48	1.87	1.85	18.79
4.759989	0.66	0.6325	15.47	1.85	1.82	19.64
4.619989	0.68	0.6624	16.03	1.82	1.80	20.39
4.48799	0.7	0.693	16.56	1.80	1.78	21.04
4.363323	0.72	0.7212	17.07	1.77	1.75	21.61
4.245395	0.74	0.75	17.54	1.74	1.73	22.10
4.133675	0.76	0.7784	17.98	1.72	1.70	22.51
4.027683	0.78	0.8063	18.39	1.69	1.68	22.85
3.926991	0.8	0.8337	18.77	1.66	1.65	23.12
3.831211	0.82	0.8606	19.11	1.63	1.63	23.33
3.739991	0.84	0.887	19.41	1.60	1.60	23.48
3.653015	0.86	0.9128	19.68	1.57	1.58	23.58
3.569992	0.88	0.938	19.91	1.54	1.55	23.62
3.490659	0.9	0.9626	20.10	1.51	1.53	23.61
3.414775	0.92	0.9867	20.26	1.48	1.50	23.55
3.34212	0.94	1.0102	20.39	1.46	1.47	23.46
3.272492	0.96	1.0331	20.47	1.43	1.45	23.32
3.205707	0.98	1.0554	20.52	1.40	1.42	23.13
3.141593	1	1.0773	20.54	1.37	1.40	22.92
2.617994	1.2	1.2684	18.97	1.12	1.17	19.05
2.243995	1.4	1.4215	14.83	0.92	0.97	12.97
1.963495	1.6	1.5496	8.86	0.77	0.80	5.49
1.745329	1.8	1.6613	1.61	0.65	0.67	-2.93
1.570796	2	1.7619	-6.53	0.56	0.57	-12.00
1.427997	2.2	1.8545	-15.33	0.49	0.49	-21.54
1.308997	2.4	1.941	-24.62	0.43	0.42	-31.42

1.208305	2.6	2.0228	-34.26	0.38	0.36	-41.55
1.121997	2.8	2.1006	-44.10	0.34	0.32	-51.89
1.047198	3	2.1752	-54.34	0.31	0.28	-62.37
0.981748	3.2	2.2471	-64.67	0.28	0.25	-72.98
0.923998	3.4	2.3164	-75.17	0.26	0.22	-83.68
0.872665	3.6	2.3836	-85.74	0.23	0.20	-94.46
0.826735	3.8	2.4488	-96.43	0.22	0.18	-105.31
0.785398	4	2.5123	-107.20	0.20	0.17	-116.20
0.747998	4.2	2.5741	-118.05	0.19	0.15	-127.15
0.713998	4.4	2.6344	-128.95	0.17	0.14	-138.12
0.682955	4.6	2.6934	-139.91	0.16	0.13	-149.13
0.654498	4.8	2.7511	-150.91	0.15	0.12	-160.17
0.628319	5	2.8075	-161.95	0.14	0.11	-171.23
0.604152	5.2	2.8629	-173.03	0.13	0.10	-182.31
0.581776	5.4	2.9172	176.52	0.13	0.09	-193.40
0.560999	5.6	2.9705	165.38	0.12	0.09	-204.52
0.541654	5.8	3.0228	154.22	0.11	0.08	-215.64
0.523599	6	3.0742	143.03	0.11	0.08	-226.78
0.506708	6.2	3.1249	131.84	0.10	0.07	-237.92
0.490874	6.4	3.1746	120.62	0.10	0.07	-249.08
0.475999	6.6	3.2237	109.38	0.09	0.06	-260.24
0.461999	6.8	3.272	98.13	0.09	0.06	-271.41
0.448799	7	3.3196	86.87	0.09	0.06	-282.58
0.436332	7.2	3.3665	75.79	0.08	0.05	-293.76
0.42454	7.4	3.4128	64.30	0.08	0.05	-304.95
0.413367	7.6	3.4584	53.01	0.08	0.05	-316.14
0.402768	7.8	3.5035	41.70	0.07	0.05	-327.34
0.392699	8	3.548	30.38	0.07	0.04	-338.54
0.383121	8.2	3.5919	19.06	0.07	0.04	-349.74
0.373999	8.4	3.6353	7.73	0.07	0.04	-360.95
0.365301	8.6	3.6782	-3.61	0.06	0.04	-372.16
0.356999	8.8	3.7206	-14.96	0.06	0.04	-383.37
0.349066	9	3.7626	-26.31	0.06	0.04	-394.58
0.341477	9.2	3.8041	-37.66	0.06	0.03	-405.80
0.334212	9.4	3.8451	-49.03	0.06	0.03	-417.02
0.327249	9.6	3.8857	-60.39	0.05	0.03	-428.24
0.320571	9.8	3.9258	-71.77	0.05	0.03	-439.46
0.314159	10	3.9656	-83.14	0.05	0.03	-450.68

1.4 Coefficients

1.4.1 Drag Coefficients

The default drag coefficient is 0.7.

Drag coefficients may be specified by two methods

1) Drag coefficients may be specified for individual elements. This input overrides any information given in alternative 2)

2) Drag coefficients are specified as function of depth

   A possible depth profile is illustrated in Figure 1.21. Values are given at grid points at various depths. The depth is specified according to a z- coordinate system, pointing upwards and with origin at mean sea surface level.

Tabulated values are taken from the table according to the element mid point. For intermediate depths values are interpolated. If member coordinate is outside the table values, the drag coefficient is extrapolated.

Because wave elevation is taken into account, drag coefficient should be given up to the maximum wave crest.

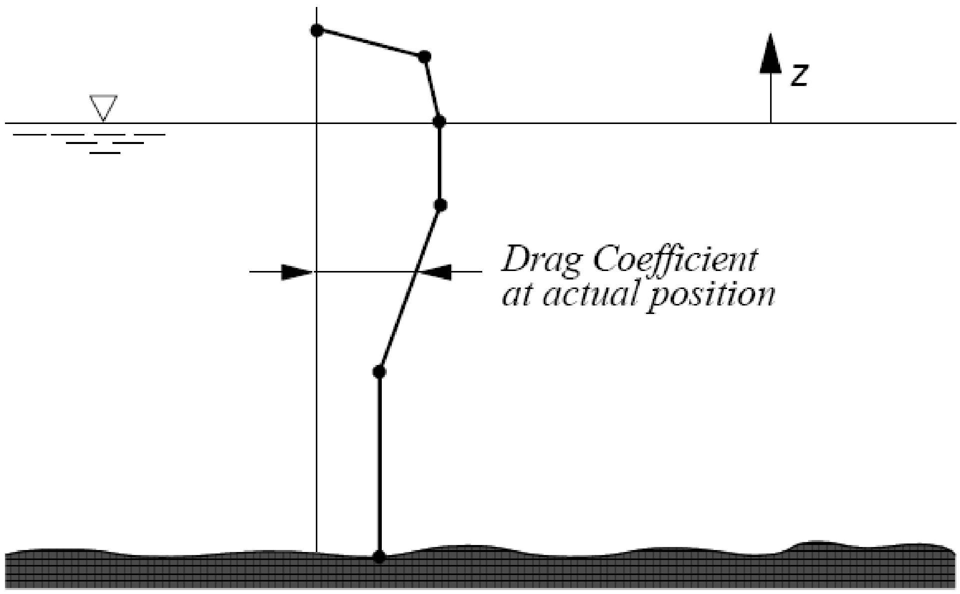


Figure 1.21 Depth profile for drag coefficient

1.4.2 Mass Coefficients

The default drag coefficient is 2.0.

Mass coefficients may be specified by two methods

1) Mass coefficients may be specified for individual elements. This input overrides any information given in alternative 2)

2) Mass coefficients are specified as function of depth

   A possible depth profile is illustrated in Figure 1.21. Values are given at grid points at various depths. The depth is specified according to a z- coordinate system, pointing upwards and with origin at mean sea surface level.

   Tabulated values are taken from the table according to the element mid point. For intermediate depths values are interpolated. If member coordinate is outside the table values, the massg coefficient is extrapolated.

Because wave elevation is taken into account, mass coefficient should be given up to the maximum wave crest.

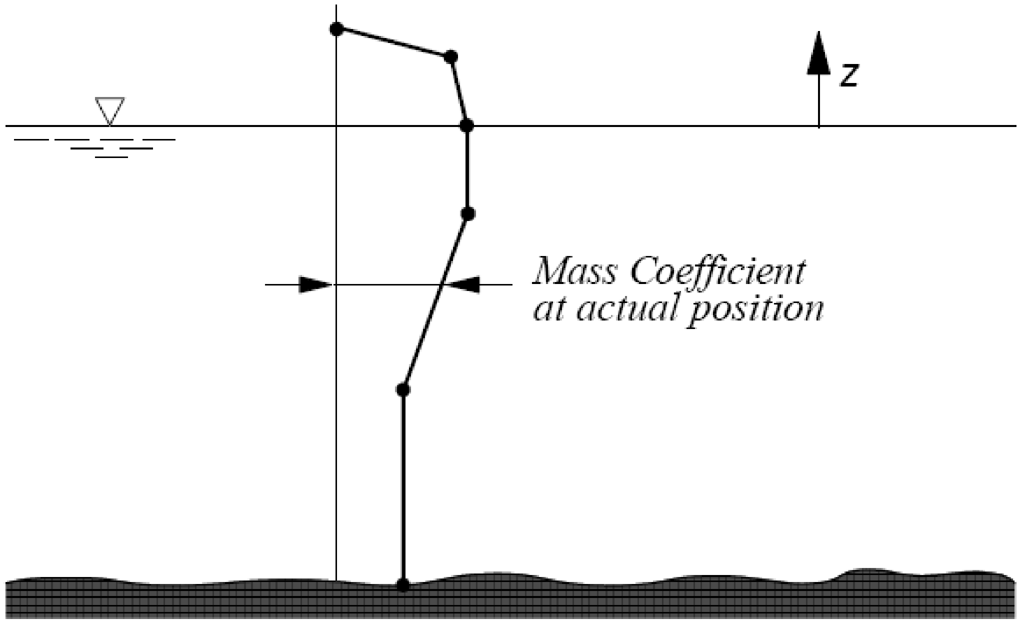


Figure 1.22 Mass profile for drag coefficient

1.5 Buoyancy

The buoyancy force may be calculated either by determination of the displaced volume ("Archimedes" force) or by direct integration of the hydrostatic - and hydrodynamic pressure over the wetted surface.

1.5.1 Archimedes

The buoyancy force of submerged members is calculated as the force of the displaced volume of the element. The position of the members relative to the sea current sea elevation is calculated so that the buoyancy force will vary according to the actual submersion.

1.5.2 Pressure integration

By this option denoted "BUOYFORM PANEL" the resultant buoyancy force is obtained by integrating the hydrodynamic and – static pressure i over the over the surface of the member. The resultant of integrating the *hydrostati*c pressure is the Archimdes buoyancy force, which is constant as long as the structure is fully immersed.

Integration of the *hydrodynamic* pressure gives a reduced buoyancy effect during a wave crest and an increase of the buoyancy during a wave trough compared to the "Archimedes" (static force) force.

The resultant of integrating the hydrodynamic pressure is equal to the mass force in Morrison's equation with C_M = 1.0. Hence, if hydrodynamic pressure is used in combination with Morrison's equation the mass force coefficient is reduced, i.e. $C_M \equiv C_M - 1$.

A problem arises when pressure integration is used. According to extrapolated Airy wave theory, the dynamic pressure is constant above mean sea level. This yields zero vertical force, which is not consistent with the mass term in Morrison's equation (non zero).

Furthermore, in the wave trough surface the resultant pressure may be substantially different from zero, which generates significant force spikes when cross-sections are only partly submerged (one side pressure) during exit or entry of water.

The remedy for this situation is to use stretched Airy theory, which always satisfies zero pressure at sea surface. Consequently, dynamic pressure according to stretched Airy theory is used regardless of whether extrapolated or stretched Airy wave theory is used otherwise. Because Morrison's theory as such is more consistent with extrapolated Airy theory, integration of the hydrodynamic pressure yields forces deviating slightly from the Morrion's mass force with C_M = 1.0

1.6 Internal Fluid

1.6.1 Flooded members

1.6.2 Free surface calculation

1.7 Marine Growth

Marine growth is specified as a thickness addition to element diameter. It may be specified by a depth profile, t_{mg} (z). The thickness of the marine growth is based upon the mid point coordinate, z_m, of the member, see also Section 1.1.

1.7.1 Modified hydrodynamic diameters

Net hydrodynamic diameter is assumed either equal to the tube diameter or as specified by input:

$$D_{hudro\_net} = \begin{matrix} D_o \\ D_{hydo\_net} \end{matrix}$$

The hydrodynamic diameter for wave force calculation according to Morrison's equation is given by

$$D_{hydro} = D_{hydro\_net} + 2t_{mg} \tag{1.77}$$

where t_{mg} is the marine growth thickness.

The calculation of drag forces is based upon the same hydrodynamic diameters, i.e.

$D_{drag} = D_{hydro}$ Diameter for drag force calculation

$D_{mass} = D_{hydro}$ Diameter for mass force calculation

1.7.2 Weight

Marine growth is characterised by its density ρ_{mg}. The mass intensity is determined by the formula

$$\rho_{mg} \frac{\pi}{4}\left(\left(D_{hydro\_net} + 2t_{mg}\right)^2 - D^2_{hydro\_net}\right) \tag{1.78}$$

When the pipe is submerged, buoyancy counteracts marine growth. If the pipe is free of water, the buoyancy disappears and the weight of the marine growth becomes "fully effective".

1.8 Quasi static wave analysis

Quasi-static wave analysis is typically carried out by incrementing a wave (and current, wind) load vector up to ultimate resistance of the structure (pushover analysis). The position of the wave which gives the largest wave action should be selected. This is done by the wave stepping option

1.8.1 Search for maxima

The maximum wave action is determined using the MAXWAVE option. The wave is stepped through the structure, refer Figure 1.23, and the wave loads corresponding to the largest action are used as the wave load vector in pushover analysis. The maximum wave action may be determined form two principles:

1) Maximum base shear
2) Maximum overturning moment

The user specifies the time increment for wave stepping

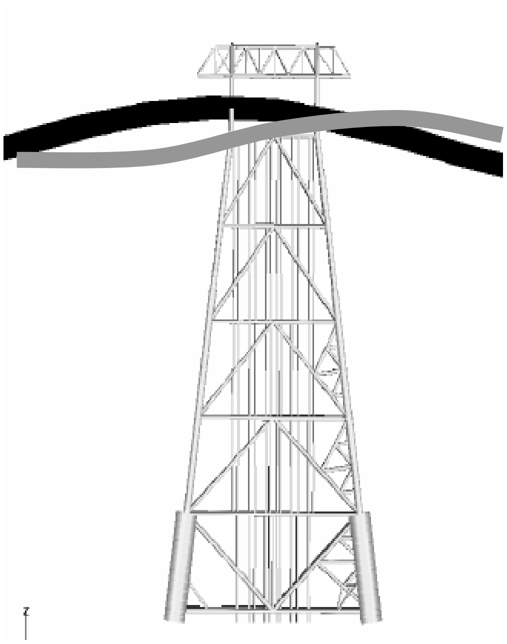


Figure 1.23 Wave stepping

2. DESCRIPTION OF USE

2.1 Hydrodynamic Parameters

HYDROPAR KeyWord Value List Type {Id_List} ……		
Parameter	*Description*	*Default*
KeyWord	Keyword defining actual parameter to define: *KeyWord* *Actual Definitions of "Value"* *HyDiam* : Hydrodynamic Diameter (override default) *Cd* : Drag Coefficient *Cm* : Mass Coefficient *Cl* : Lift Coefficient (not imp) *BuDiam* : Buoyancy Diameter (override default) *IntDiam* : Internal Diameter (override default) *WaveInt* : Number of integration points (override default) *CurrBlock* : Current Blockage factor *FluiDens* : Density of internal fluid *MgrThick* : Marine Growth Thickness *MgrDens* : Marine Growth Density *FloodSW* : Switch for flooded/no flooded (override default) *DirDepSW* : Switch for use of direction dependent Cd *FillRatio* : Fill ratio, (0-1) of member with internal fluid *WaveKRF* : Wave Kinematics Reduction Coeff *BuoyLevel* : Definition of complexity level of buoyancy calculations.	
Value	Actual Parameter value.	
ListTyp	Data type used to specify the element(s): *Element* : The specified Id's are element numbers. *Mat* : The specified Id's are material numbers *Geo* : The specified Id's are geometry numbers. *Group* : The specified Id's are group numbers.	
Id_List	One or several id's separated by space	

With this record, the user defines various hydrodynamic parameters for elements. Some of the parameters could be defined using alternative commands (F ex Hyd_CdCm etc), but parameters defined under HYDROPAR will *override all previous definitions*.

This record could be repeated

Below, the "HYDROPAR" keywords are described in detail:

HYDROPAR Keyword …….

Keyword	Description	Default
HyDiam	The hydrodynamic diameter is used in connection with drag- and mass forces according to Morrison's equation.	Struct Do
Cd / Cm	Drag- and Mass coefficients used in Morrison's equation.	0.7 / 2.0
Cl	Lift coefficients (normal to fluid flow). NOTE: Not implemented hydro.	0
BuDiam	Buoyancy calculations are based on this diameter.	Struct D
IntDiam	Internal diameter of the pipe. Relevant in connection with (completely) flooded members and members with special internal fluid.	Do-2T
WaveInt	Number of integration points per element	2
CurrBlock	Current blockage factor. Current is multiplied with this factor.	1.0
FluiDens	Density of *internal* fluid. Relevant for flooded members.	1024
MgrThick	Thickness of marine growth specified in meter.	0.0
MgrDens	Density of marine growth. Specified in [kg/m^3]	1024
FloodSW	Switch (0/1) for flooded / non flooded members. (internal use)	0
DirDepSW	Switch (0/1) for use of direction dependent drag coefficients. If switch is set to 1, special ElmCoeff data have to be defined for the element.	0
FillRatio	Fill ratio of flooded member. By default is a flooded member 100% filled throughout the simulation. Fill ratio could be time dependent.	1
WaveKRF	Wave kinematics reduction coefficient. Particle velocity used for actual elements is multiplied with this factor.	1.0
BuoyLevel	Specification of buoyancy calculation method. By default, the buoyancy of the (thin) steel wall is ignored for flooded members. If Level=1 is specified, a far more complex (and time consuming) calculation procedure is used. Flooded members on a floating structure going in and out of water should use Level=1 calculation.	0

Wave_Int *Profile* Z_1 $nInt_1$
Z_2 $nInt_2$
... ...
Z_n $nInt_n$

Parameter	*Description*	*Default*
Z_1	Z-coordinate of the first grid point defining the Integration Point profile (Z=0 defines the sea surface, and all Z-coordinates are given relative to the surface, Z-axis is pointing upwards. Z>0 means *above* the sea surface).	
$nInt_1$	Number of Integration Points to be used for elements at position Z_1	
Z_2	Z-coordinate of the second grid point.	
$nInt_2$	Number of Integration Points to be used for elements at elevation Z_2	

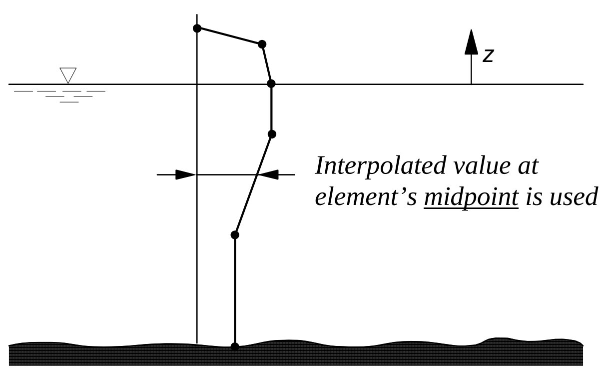
This record is used to define a **Integration Point depth profile**, and is an extended version of the original *Wave_Int* command.

Between the tabulated values, the nInt is interpolated. Values outside the table are *extrapolated*.

In the .out -file, the interpolated number of integration points used for each beam element is listed. Selected values are also visualized in XACT under Verify/Hydrodynamics.

Data should also be specified *above* the sea surface. Ensure that extrapolation gives correct *nInt*, (dry elements become wet due to surface wave elevation).

The command "`HYDROPAR` *`WaveInt`* " overrides this command.



NOTE! **SI units** must be used (N, m, kg) with **Z-axis pointing upwards!**

This record is given only once.

CurrBlock *Profile* Z_1 $Block_1$
Z_2 $Block_2$
... ...
Z_n $Block_n$

Parameter	*Description*	*Default*
Z_1	Z-coordinate of the first grid point defining the Integration Point profile (Z=0 defines the sea surface, and all Z-coordinates are given relative to the surface, Z-axis is pointing upwards. Z>0 means *above* the sea surface).	
$Block_1$	Current Blockage to be used for elements at position Z_1	
Z_2	Z-coordinate of the second grid point.	
$Block_2$	Current Blockage to be used for elements at elevation Z_2	

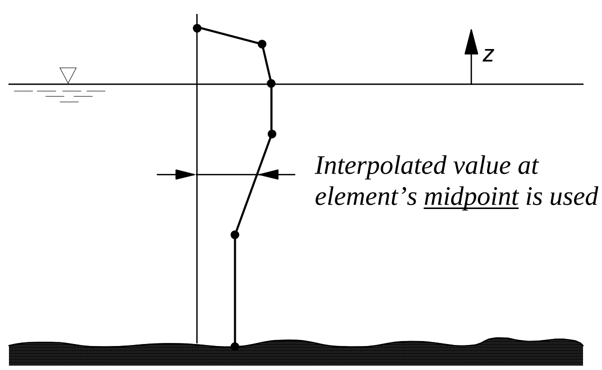
This record is used to define a **Current Blockage depth profile**, and is an extended version of the original *CurrBlock* command.

Between the tabulated values, the Block value is interpolated. Values outside the table are *extrapolated*.

In the .out -file, the interpolated blockage factor used for each beam element is listed. Selected values are also visualized in XACT under Verify/Hydrodynamics.

Data should also be specified *above* the sea surface. Ensure that extrapolation gives correct Block, (dry elements become wet due to surface wave elevation).

The command “`HYDROPAR` *`CurrBlock`* “ overrides this command.



NOTE! **SI units** must be used (N, m, kg) with **Z-axis pointing upwards!**

This record is given only once.

Wave_KRF	*Profile*	Z_1	KRF_1
		Z_2	KRF_2
		...	...
		Z_n	KRF_n

Parameter	*Description*	*Default*
Z_1	Z-coordinate of the first grid point defining the Integration Point profile (Z=0 defines the sea surface, and all Z-coordinates are given relative to the surface, Z-axis is pointing upwards. Z>0 means *above* the sea surface).	
KRF_1	Kinematics Reduction Factor to be used for elements at position Z_1	
Z_2	Z-coordinate of the second grid point.	
KRF_2	Kinematics Reduction Factor to be used for elements at elevation Z_2	

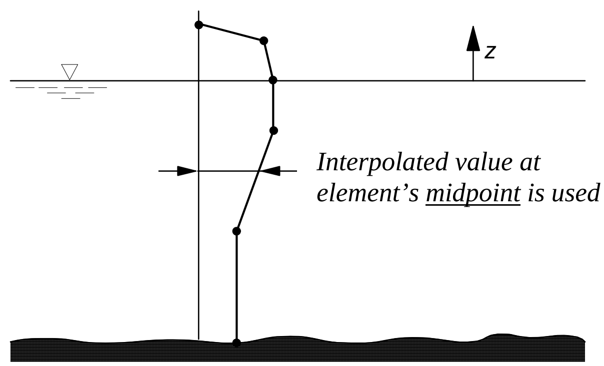
This record is used to define a **Kinematics Reduction Factor depth profile**, and is an extended version of the original *Wave_KRF* command.

Between the tabulated values, the KRF is interpolated. Values outside the table are *extrapolated*.

In the .out -file, the interpolated wave kinematics reduction factor used for each beam element is listed.
Selected values are also visualized in XACT under Verify/Hydrodynamics.

Data should also be specified *above* the sea surface. Ensure that extrapolation gives correct KRF, (dry elements become wet due to surface wave elevation).

The command "`HYDROPAR` *`WaveKRF`* " overrides this command.



NOTE! **SI units** must be used (N, m, kg) with **Z-axis pointing upwards!**

This record is given only once.

2.2 Waves

WAVEDATA l_case Type Height Period Direct Phase Surflev Depth	N_ini X_1 f_1 X_2 f_2 ... ... X_n f_n	
Parameter	*Description*	*Default*
l_case	Load case number. The wave is activated by using the LOADHIST command referring to this load case number + a TIMEHIST of type 3	
Type	Wave Type 1 : Airy, Extrapolated 1.1 : Airy, Stretched 2 : Stoke's 5'th (Skjelbreia, Hendrickson, 1961) 3 : User Defined 4 : Stream Function Theory (Dean, Dalrymple)	
	Unit	
Height	Wave height [m]	
Period	Wave period [s]	
Direct	Direction of wave relative to global x-axis, counter clockwise [dg]	
Phase	Wave phase [dg]	
Surflev	Surface Level (Z-coordinate) expressed in global system [m]	
Depth	Water depth [m]	
n_ini	Number of initialisation points defining wave 'envelope'	0
X_1	X-coordinate of first grid point (starting with largest negative x-coord.)	
f_1	Scaling factor of the wave height at first grid point, see Figure 2.1	

With this record, the user may specify a wave to be applied to the structure as hydrodynamic forces.
The wave is 'switched' ON according to the LOADHIST/TIMEHIST definition. TIMEHIST **type 3 must** be used.

Wave forces are applied on the structural members, which are **wet at the time of load calculation**, and relative velocity is accounted for if the record REL_VELO is specified in the control file.

Current to be combined with the actual wave **must have same load case number!**
Doppler effect is included.

Time between calculation of wave forces is controlled by the referred TIMEHIST record, (dTime).
The calculated wave forces are written to file if WAVCASE1 is specified in the control file.

In XACT the surface elevation is visualized.

NOTE! **SI units** must be used (N, m, kg) with **Z-axis pointing upwards!**
This record may be repeated

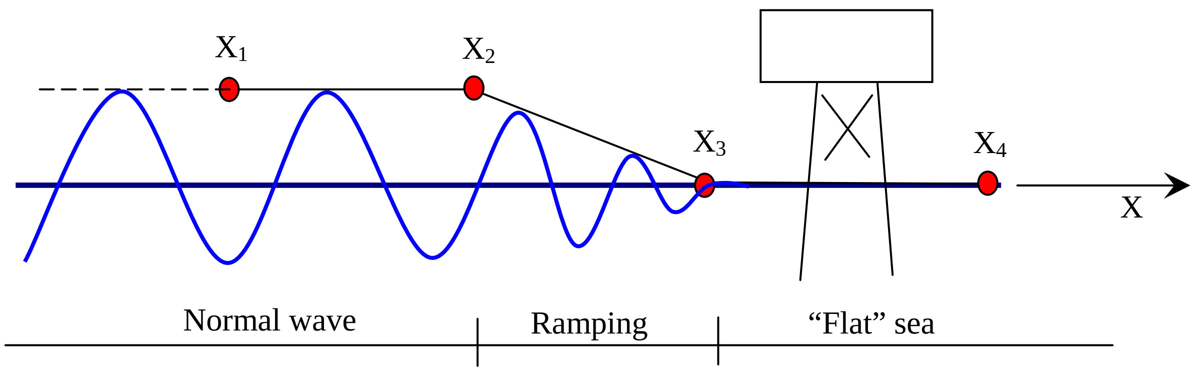


Figure 2.1 Initialisation of wave

WAVEDATA Lcase Type Hs Tp Direct Seed Surflev Depth N_ini
X_1 f_1
X_n f_n
nFreq SpecType TMin TMax Grid (Opt) {Data}

Parameter	*Description*		*Default*
l_case	Load case number. The wave is activated by using the LOADHIST command referring to this load case number + a TIMEHIST of type 3		
Type	Wave Type = *Spect*	Unit	
Hs	Significant Wave height	[m]	
Tp	Peak period of spectre	[s]	
Direct	Direction of wave relative to global x-axis, counter clockwise	[dg]	
Seed	Wave seed (input to random generator)	[-]	
Surflev	Surface Level (Z-coordinate) expressed in global system	[m]	
Depth	Water depth	[m]	
n_ini	Number of initialisation points defining wave 'envelope (see previous page).		0
nFreq	Number of frequencies		
SpecType	Specter type. Jonswap : Jonwap Spectre PM : Pierson-Moscovitz User : User Defined Spectre		
TMin	Lowest wave period to be used in the wave representation		
TMax	Highest wave period to be used.		
Grid	Discretization type: 1 : Constant dω in the interval tmin-tmax 2 : Geometrical series from Tp 3 : Constant area for each S(ω) "bar"		2
"Opt"	Optional Data. If Jonswap : Gamma parameter If User Defined : Number of points in the ω – S(ω) curve Else : Dummy		
{Data}	If User Defined : The nPoint ω – S(ω) points defining S(ω) Else : Dummy		

With this record, the user may specify an irregular wave to be applied to the structure as hydrodynamic forces as described on the previous page.

Example:

```
          LCase  Typ    Hs     Tp    Dir  Seed SurfLev  Depth  nIni
 WaveData  3   Spect  12.8  13.3   45   12       0      176     0
'          nFreq    SpecTyp  TMin   TMax   (Grid)
             30      Jonsw    4      20
```

See the example collection on www.usfos.com for more examples.

This record is given once.

2.3 Current

CURRENT l_case Speed Direct Surflev Depth Z_1 f_1
Z_2 f_2
... ...
Z_n f_n

Parameter	*Description*		*Default*
l_case	Load case number. The current is activated by using the LOADHIST command referring to this load case number + a TIMEHIST of type 3		
		Unit	
Speed	Current Speed to be multiplied with the factor f giving the speed at actual depth, (if profile is defined)	[m/s]	
Direct	Direction of wave relative to global x-axis, counter clockwise	[deg]	
Surflev	Surface Level (Z-coordinate) expressed in global system	[m]	
Depth	Water depth	[m]	
Z_1	Z-coordinate of first grid point (starting at Sea Surface)	[m]	
f_1	Scaling factor of the defined *speed* at first grid point.		
....	Similar for all points defining the **depth profile** of the current		

With this record, the user may specify a current to be applied to the structure as hydrodynamic forces.
The current is 'switched' ON according to the LOADHIST/TIMEHIST definition. TIMEHIST **type 3 must** be used. If the current should vary over time, the CURRHIST command is used.

Wave forces are applied on the structural members which are **wet at the time of load calculation**, and relative velocity is accounted for if the record REL_VELO is specified in the control file.

Current to be combined with waves **must have same load case number!**

Time between calculation of wave forces is controlled by the referred TIMEHIST record, (dTime). The calculated wave forces are written to file if WAVCASE1 is specified in the control file.

In XACT the surface elevation is visualised. Applying a mesh on the surface (Verify/Show mesh) the waves become clearer, (Result/deformed model must be activated with displacement scaling factor=1.0). By pointing on the sea surface using the option Clip/Element, the surface will disappear.

NOTE! SI units must be used (N, m, kg) with **Z-axis pointing upwards!**

This record may be repeated

3. VERIFICATION

In the present chapter hydrodynamic kinematics and – forces simulated by USFOS are compared with results form alternative calculations with Excel spreadsheet and Visual Basic Macros, developed for verification purposes.

The verification comprises the following tasks:

- Drag force due to current only
- Airy wave kinematics deep water (depth 20 m)
- Airy wave kinematics finite water depth (depth 20 m)
- Stokes wave kinematics – wave height 30 (depth 70 m)
- Stokes wave kinematics – wave height 33 (depth 70 m)
- Comparison of Stokes and Dean wave kinematics – wave height 30 m and 36 m (depth 70 m)
- Wave forces on oblique pipe, 20 m water depth – Airy deep water theory
- Wave forces on oblique pipe, 20 m water depth – Airy finite depth theory
- Wave and current forces on oblique pipe, 20 m water depth- Stokes theory
- Wave forces on vertical pipe, 70 m water depth – Airy finite depth theory
- Wave forces on vertical pipe, 70 m water depth – Stokes theory
- Wave forces on oblique pipe, 70 m water depth – Stokes theory
- Wave forces on oblique pipe, 70 m water depth , different wave direction– Stokes theory
- Wave and current forces on oblique pipe, 70 m water depth– Stokes theory
- Wave and current forces on oblique pipe, 10 elements , 70 m water depth– Stokes theory
- Wave and current forces on oblique pipe, relative velocity , 70 m water depth– Airy theory
- Wave and current forces on oblique pipe, relative velocity , 70 m water depth– Stokes theory
- Wave and current forces on oblique pipe, relative velocity , 70 m water depth– Dean's theory
- Buoyancy forces

The actual values used in the calculation are tabulated for each case.

A vertical pipe is located with the lower end at sea floor and the upper end above crest height. It runs parallel to the z-axis.

An oblique pipe is located with the lower end at sea floor and the upper end above crest height. It is running in three dimensions.

The horizontal pipe runs parallel to the sea surface and is partly above sea elevation.

In order to minimize discretization errors the pipe is generally subdivided into 100 elements. The default value of 10 integration points along each element is used.
In the most general case the wave direction, the current direction and the structure motion direction are different and do not coincide with the pipe orientation.

The diameter of the pipe is 0.2 m. For the chosen diameter drag forces will dominate the wave action. In order to allow proper comparison, the drag force and mass force are calculated separately. The drag – and mass force coefficients used are tabulated in each case. In most cases C_D = 1.0 and C_M = 2.0.

Airy – and Stokes kinematics and forces are compared with spreadsheet calculations. No spreadsheet algorithm has been developed for Dean's theory. However, kinematics are compared with results from computer algorithm developed by Dalrymple. The Comparison shows that the difference between Stokes and dean's theories starts to become significant for waves higher than 30 m at 70 m water depth (period 16 seconds)

Hydrodynamic forces are calculated by means of static analysis, with the exception of the cases where relative motion has been taken into account. These cases have been simulated using a prescribed nodal velocity history. The procedure induces high frequency vibrations; however, average simulation results are close to spreadsheet calculations. The high frequency vibrations (accelerations) induced do not allow for comparison with the mass force component.

The verifications show that the wave kinematics predicted with USFOS
Agree well with spreadsheet calculations. The agreement is also very good as concerns the drag – and mass force evolution.

3.1 Current

The current speed is assumed to vary linearly with depth, with 0 m/s at sea floor and 1.5 m/s at sea surface.

	Period [s]	**Height [m]**	**Theory**
	15	0.0002	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**330**
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	30	1.5
Y	0	30	**Curr dir**
Z	-70	20	270
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	-515.432	-513.979	0.0028
X_min	-515.778	-516.361	0.0011
Y_max	5156.271	5162.840	0.0013
Y_min	5155.834	5140.540	0.0030
Z_max	-1546.777	-1542.180	0.0030
Z_min	-1546.855	-1548.840	0.0013

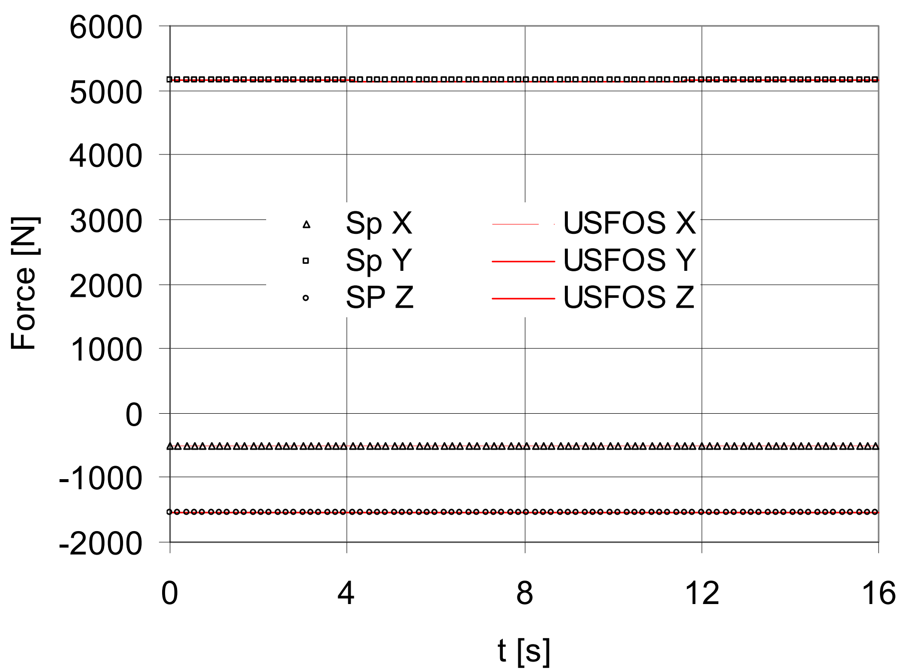


Figure 3.1 Drag force components from current only

3.2 Waves

3.2.1 Airy wave kinematics –deep water

	Period [s]	Height [m]	Theory
	5	5	Deep
	Depth [m]	Diameter [m]	
	20	0.2	
	C_d	C_m	
	1	0	
	Coord 1 [m]	Coord 2 [m]	
X	0	0	
Y	0	0	
Z	-20	7	

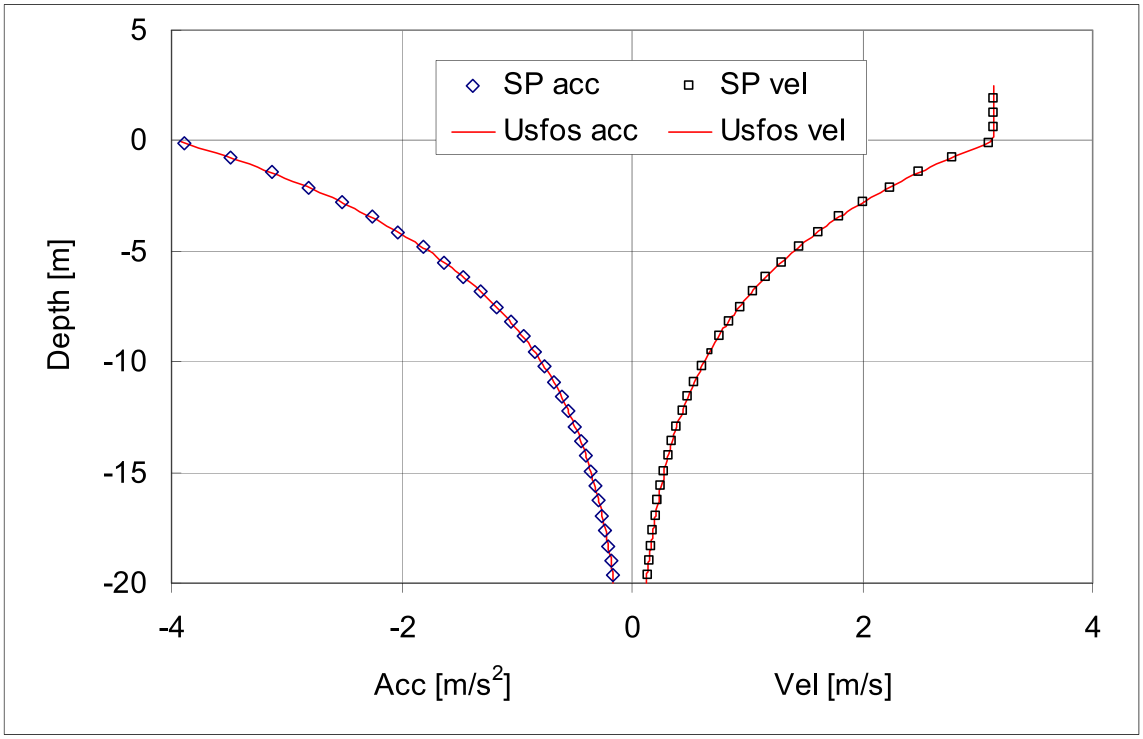


Figure 3.2 Velocity (t = 0 s) - and acceleration (t = 1.25 s) profile

3.2.2 Airy wave kinematics –finite water depth

	Period [s]	Height [m]	Theory
	8	5	Finite
	Depth [m]	Diameter [m]	
	20	0.2	
	C_d	C_m	
	1	0	
	Coord 1 [m]	Coord 2 [m]	
X	0	0	
Y	0	0	
Z	-20	7	

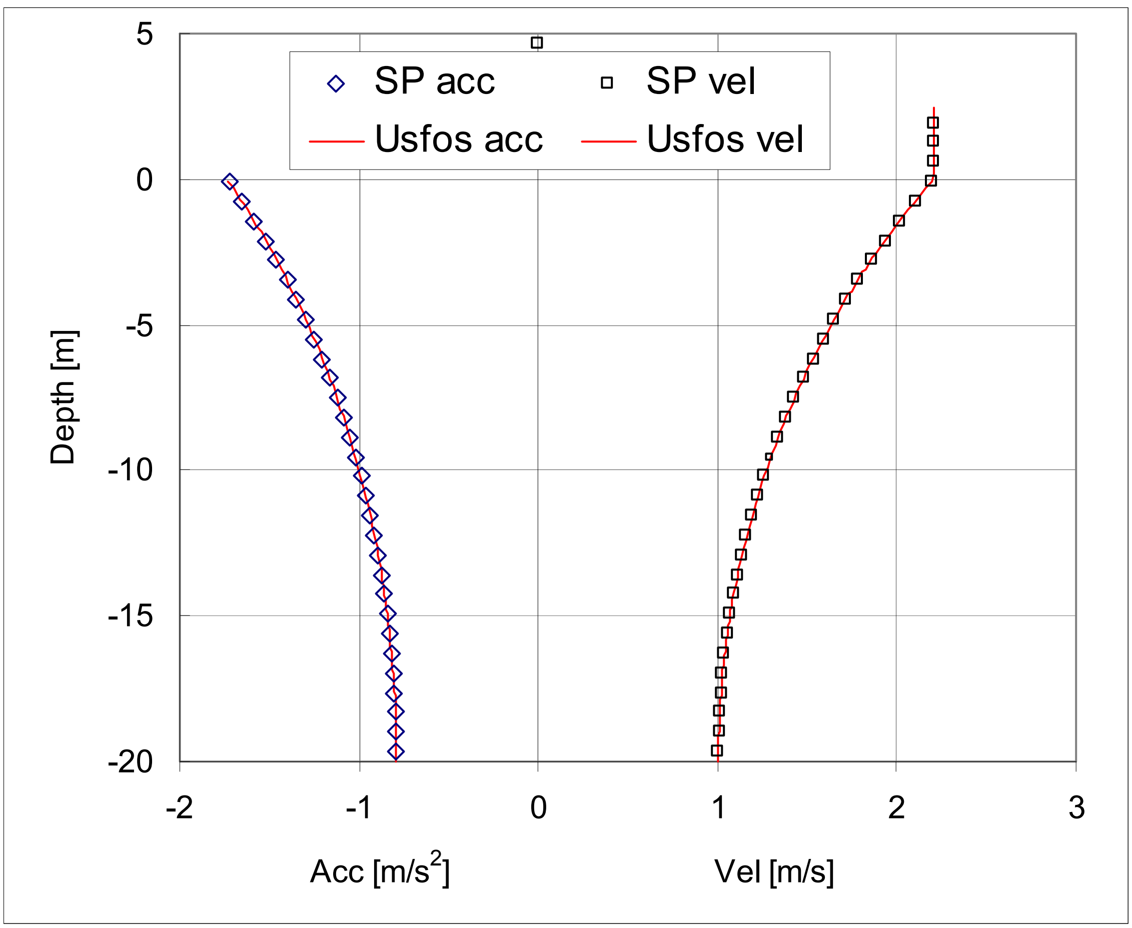


Figure 3.3 Velocity (t = 0 s) - and acceleration (t = 2.0 s) profile

3.2.3 Extrapolated Airy wave kinematics – finite water depth

Sea floor z = 5.0 m, Depth z = 85 m, Surface level z = 90 m. Wave height H = 18 m, Wave period = 14.0 seconds. Waves propagating in positive x-direction.

Figure 3.4 and Figure 3.5 show the wave particle velocity and acceleration histories calculated at two different vertical locations, close to wave trough and above mean sea surface). The usfos calculation shows that the speed and acceleration immediately becomes zero once the water level falls below the z-coordinate level (this is not taken into account in the spreadsheet calculations). Excellent agreement with spreadsheet calculations are obtained otherwise.

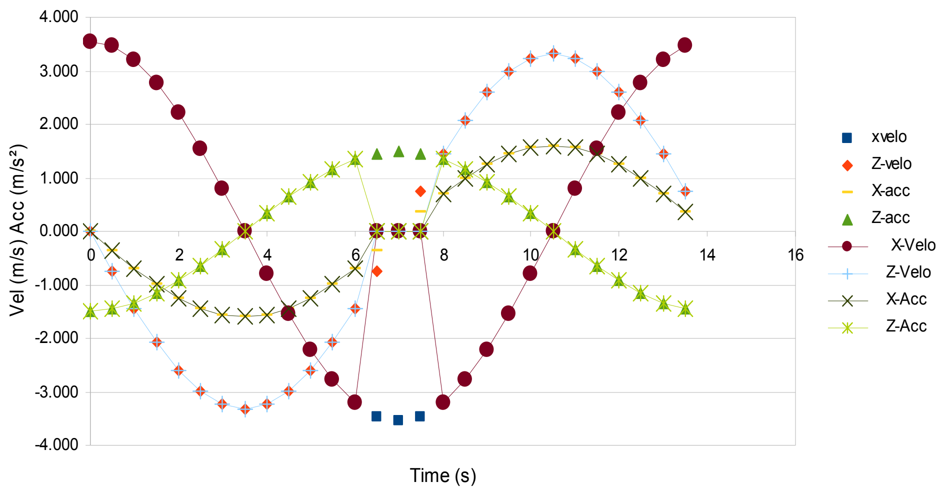


Figure 3.4 Wave particle velocity and acceleration for z = 81.5 m (close to trough). (Usfos full line, spreadsheet Markers only)

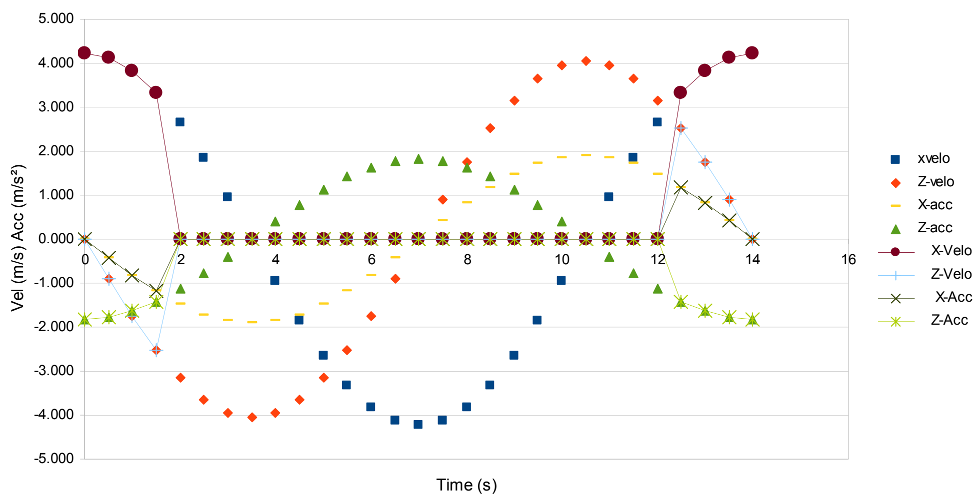


Figure 3.5 Wave particle velocity and acceleration for z = 96 m. (Usfos full line, spreadsheet Markers only)

3.2.4 Stretched Airy wave kinematics – finite water depth

Sea floor z = 5.0 m, Depth z = 85 m, Surface level z = 90 m. Wave height H = 18 m, Wave period = 14.0 seconds. Waves propagating in positive x-direction.

Figure 3.6 and Figure 3.7 show the wave particle velocity and acceleration histories calculated at two different vertical locations, close to wave trough and above mean sea surface). The usfos calculation shows that the speed and acceleration immediately becomes zero once the water level falls below the z-coordinate level (this is not taken into account in the spreadsheet calculations). Excellent agreement with spreadsheet calculations are obtained otherwise.

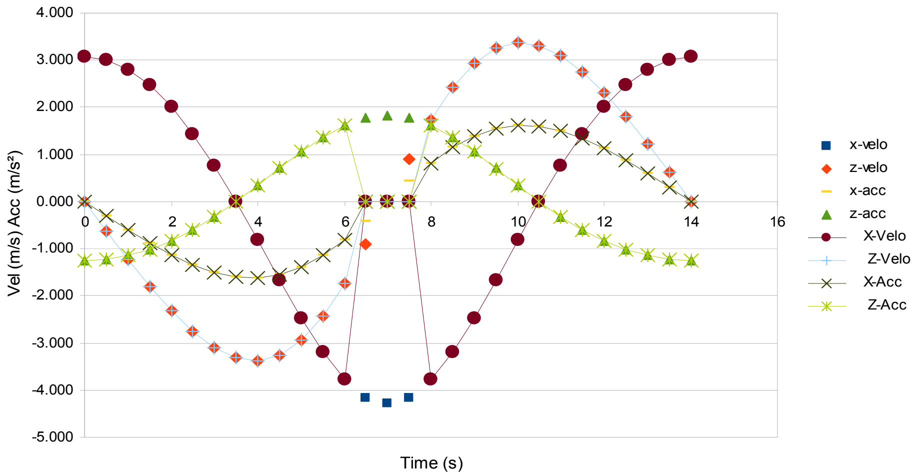


Figure 3.6 Wave particle velocity and acceleration for z = 81.5 m (close to trough). (Usfos full line, spreadsheet Markers only)

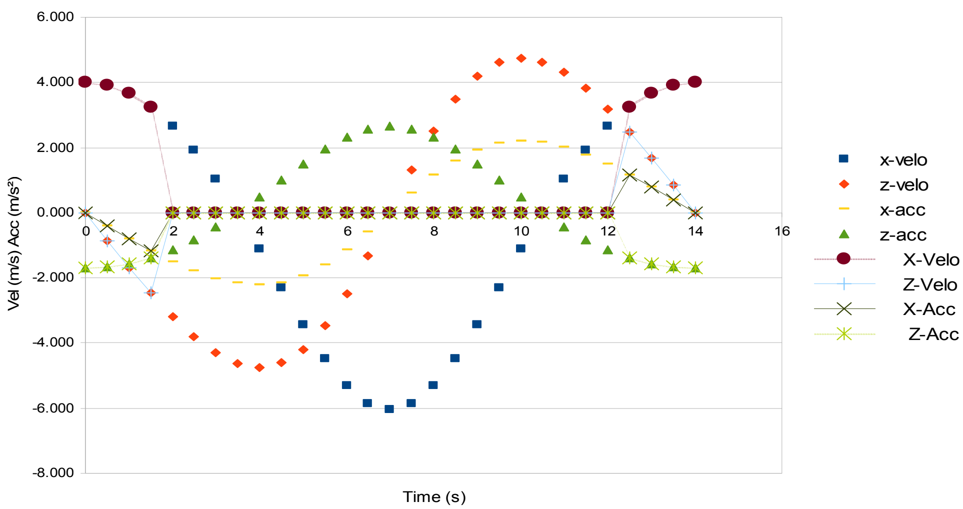


Figure 3.7 Wave particle velocity and acceleration for z = 96 m. (Usfos full line, spreadsheet Markers only)

3.2.5 Stokes wave kinematics –Wave height 30m

	Period [s]	Height [m]	Theory
	16	30	Stokes
	Depth [m]	**Diameter [m]**	
	70	0.2	
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	
X	0	0	
Y	0	0	
Z	-70	20	

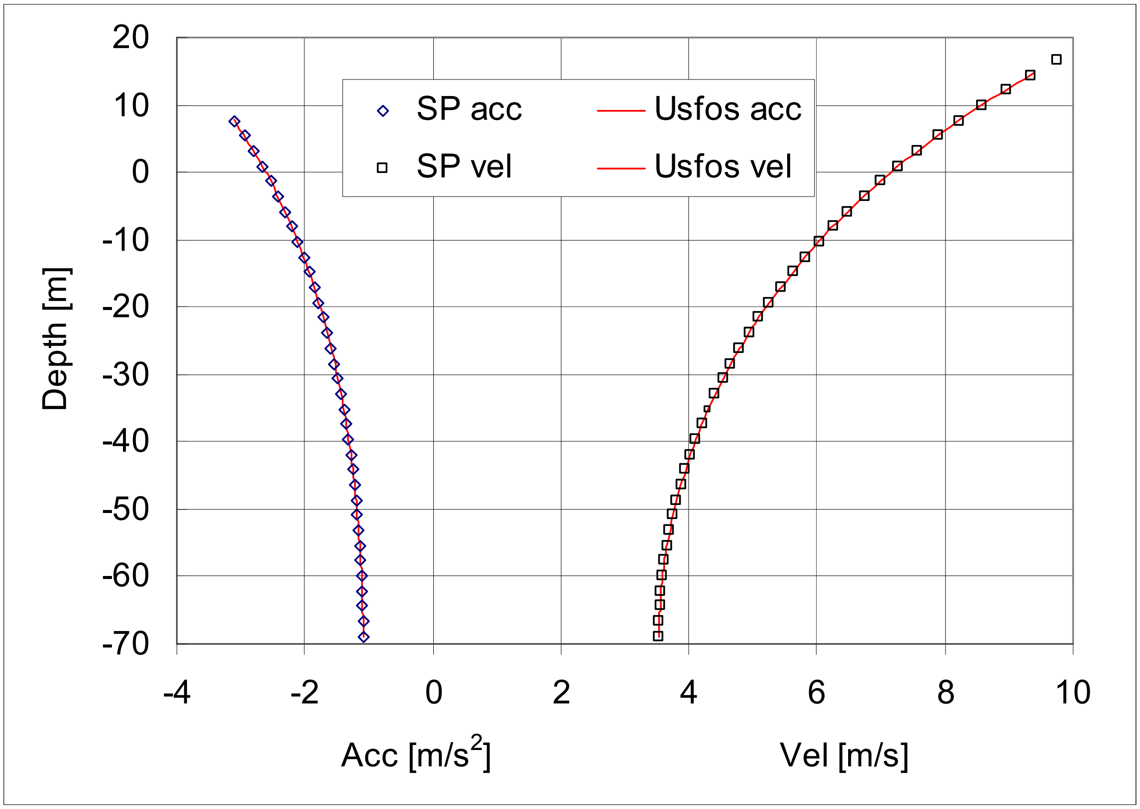


Figure 3.8 Velocity (t = 0 s) - and acceleration (t = 2.0 s) profile

3.2.6 Stokes wave kinematics –Wave height 33 m

	Period [s]	Height [m]	Theory
	16	33	Stokes
	Depth [m]	**Diameter [m]**	
	70	0.2	
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	
X	0	0	
Y	0	0	
Z	-70	20	

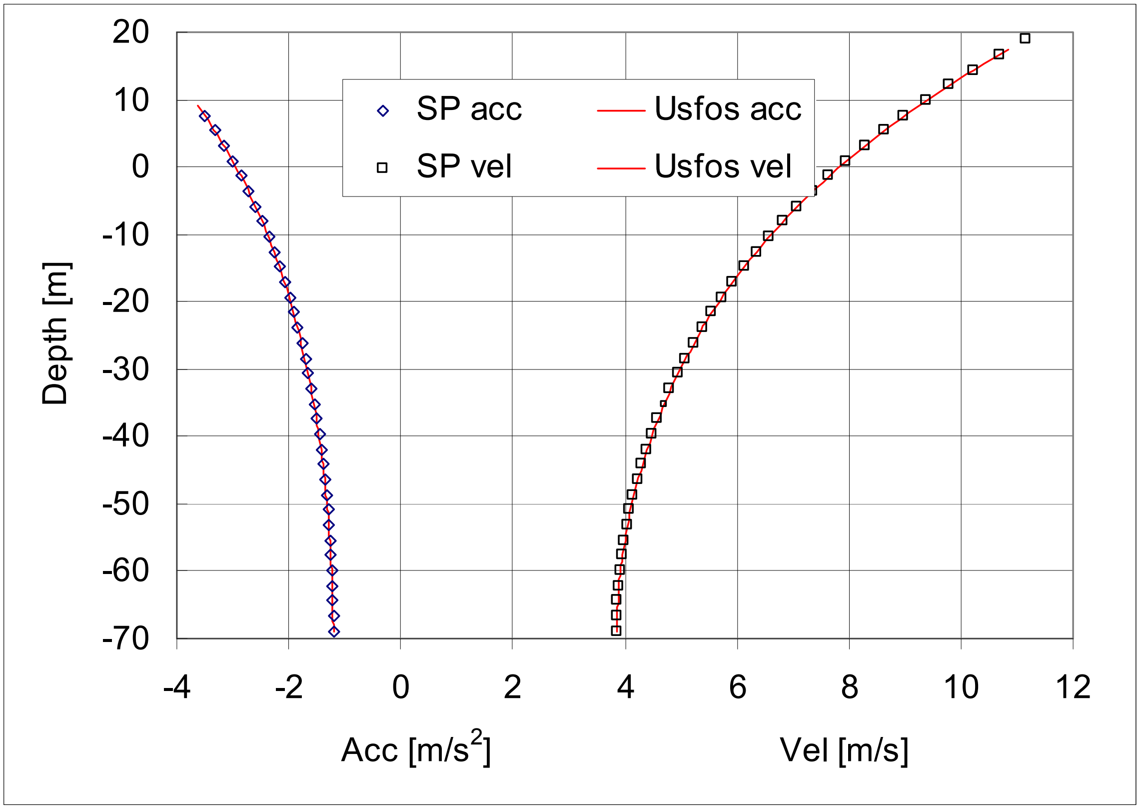


Figure 3.9 Velocity (t = 0 s) - and acceleration (t = 2.0 s) profile

3.2.7 Stokes and Dean wave kinematics –Wave height 30 and 36 m

	Period [s]	**Height [m]**	**Theories**
	16	30/36	**Stokes**
	Depth [m]	**Diameter [m]**	**Dean**
	70	0.2	
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	
X	0	0	
Y	0	0	
Z	-70	30	

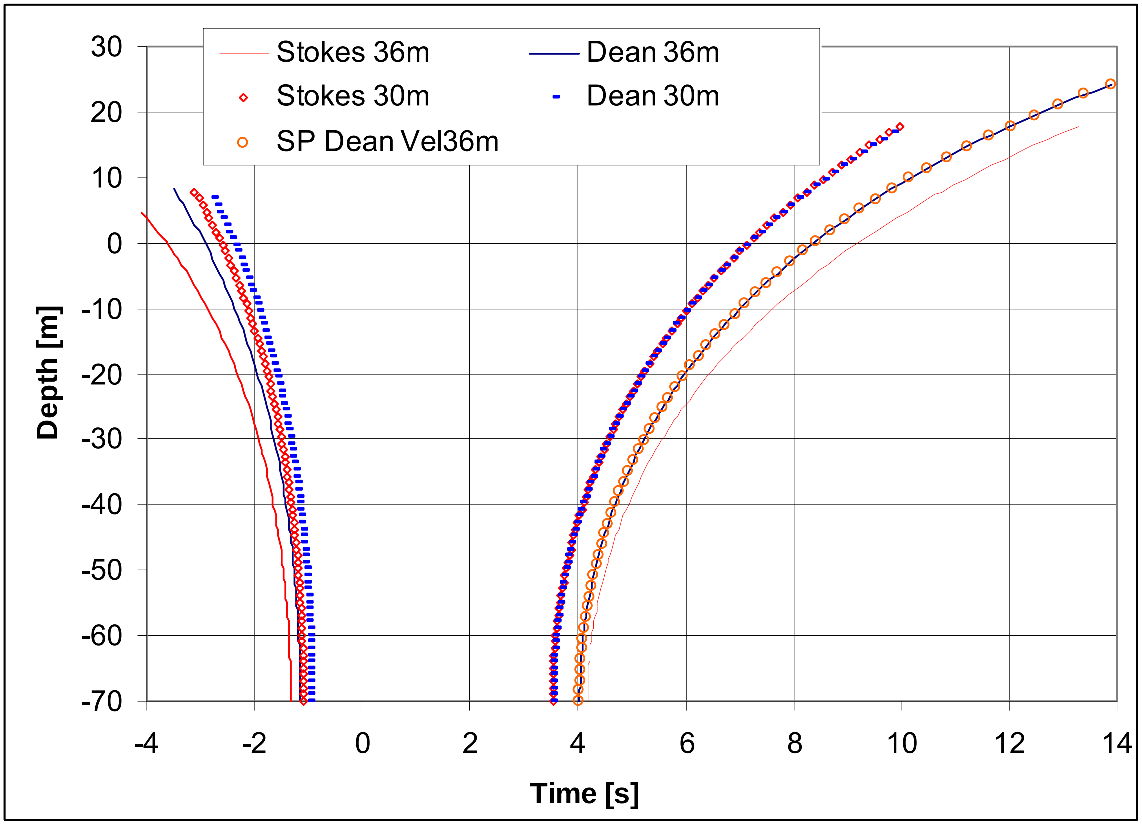


Figure 3.10 Velocity (t = 0 s) - and acceleration (t = 2.0 s) profile.

3.2.8 Wave forces oblique pipe, 20m depth – Airy deep water theory

	Period [s]	Height [m]	Theory
	5	5	Deep
	Depth [m]	Diameter [m]	
	20	0.2	
	C_d	C_m	
	1	0	
	Coord 1 [m]	Coord 2 [m]	
X	0	10	
Y	0	10	
Z	-20	7	
	Reaction SP [N]	Reaction Usfos [N]	Deviation
X_max	1467.962	1475.850	0.0053
X_min	-5788.126	-5778.270	0.0017
Y_max	903.039	905.301	0.0025
Y_min	-814.670	-813.187	0.0018
Z_max	2165.216	2162.700	0.0012
Z_min	-586.403	-590.216	0.0065

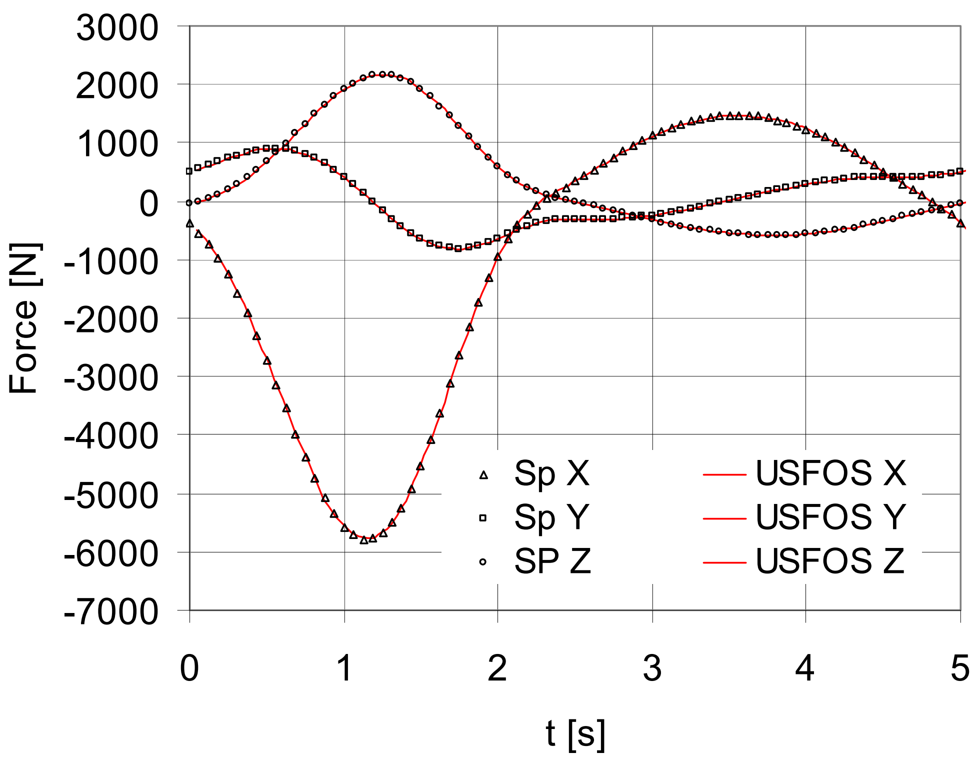


Figure 3.11 Histories of drag force components

	Period [s]	Height [m]	Theory
	5	5	Deep
	Depth [m]	Diameter [m]	Wave dir
	20	0.2	**0**
	C_d	C_m	
	0	2	
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	10	0
Y	0	10	**Curr dir**
Z	-20	7	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	1658.998	1663.440	0.0027
X_min	-1654.683	-1647.340	0.0045
Y_max	336.462	336.910	0.0013
Y_min	-719.775	-717.250	0.0035
Z_max	703.985	704.857	0.0012
Z_min	-566.730	-566.771	0.0001

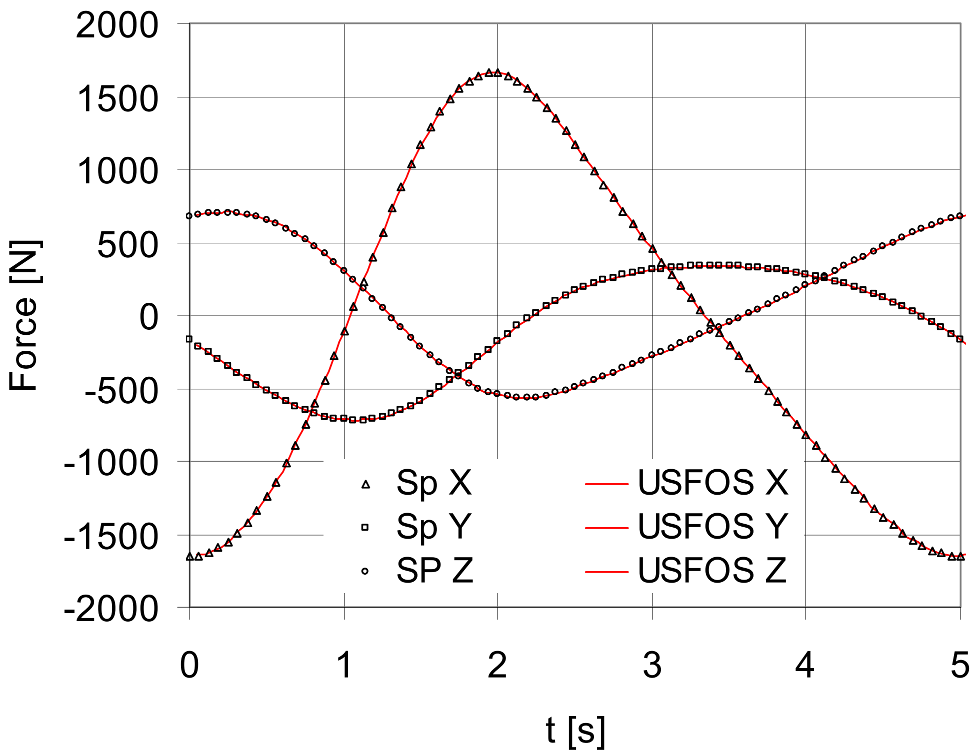


Figure 3.12 Histories of mass force components

3.2.9 Wave forces oblique pipe, 20 m depth – Airy finite depth theory

	Period [s]	Height [m]	Theory
	8	5	Finite
	Depth [m]	Diameter [m]	Wave dir
	20	0.2	0
	C_d	C_m	
	1	0	
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	10	0
Y	0	10	Curr dir
Z	-20	7	0
	Reaction SP [N]	Reaction Usfos [N]	Deviation
X_max	2941.080	2917.690	0.0080
X_min	-5189.809	-5162.070	0.0054
Y_max	827.832	826.692	0.0014
Y_min	-501.009	-501.018	0.0000
Z_max	1822.678	1819.330	0.0018
Z_min	-1012.527	-1010.230	0.0023

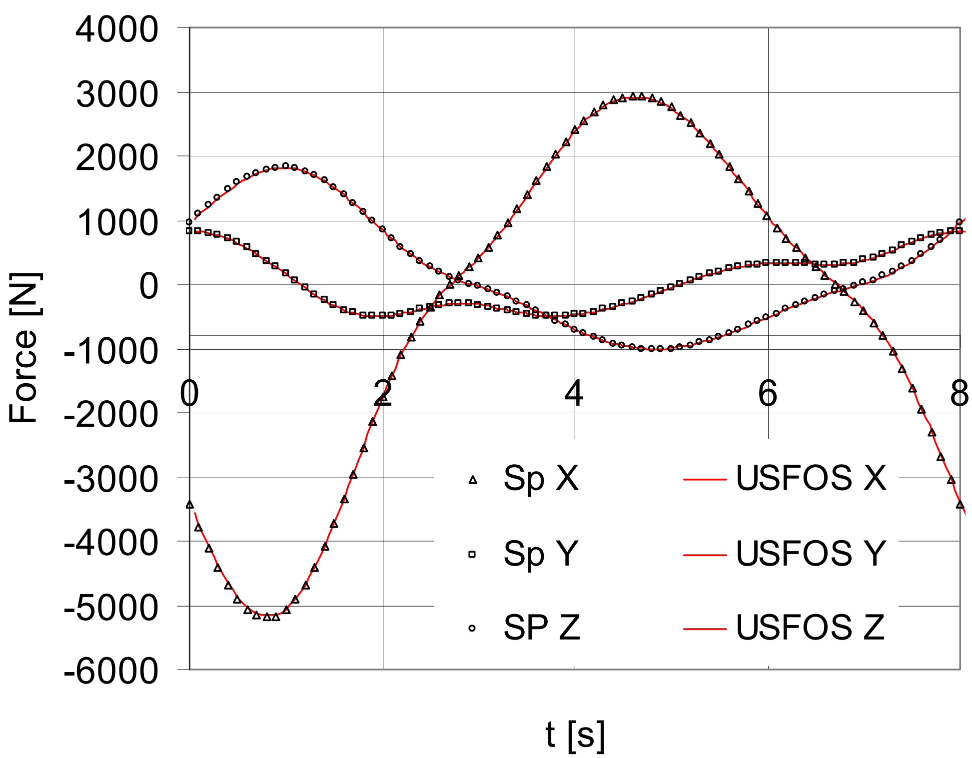


Figure 3.13 Histories of drag force components

	Period [s]	Height [m]	Theory
	8	5	Finite
	Depth [m]	Diameter [m]	Wave dir
	20	0.2	0
	C_d	C_m	
	0	2	
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	10	0
Y	0	10	Curr dir
Z	-20	7	0
	Reaction SP [N]	Reaction Usfos [N]	Deviation
X_max	1434.748	1423.310	0.0080
X_min	-1417.983	-1409.480	0.0060
Y_max	261.723	260.803	0.0035
Y_min	-417.741	-416.448	0.0031
Z_max	516.255	514.334	0.0037
Z_min	-474.139	-471.545	0.0055

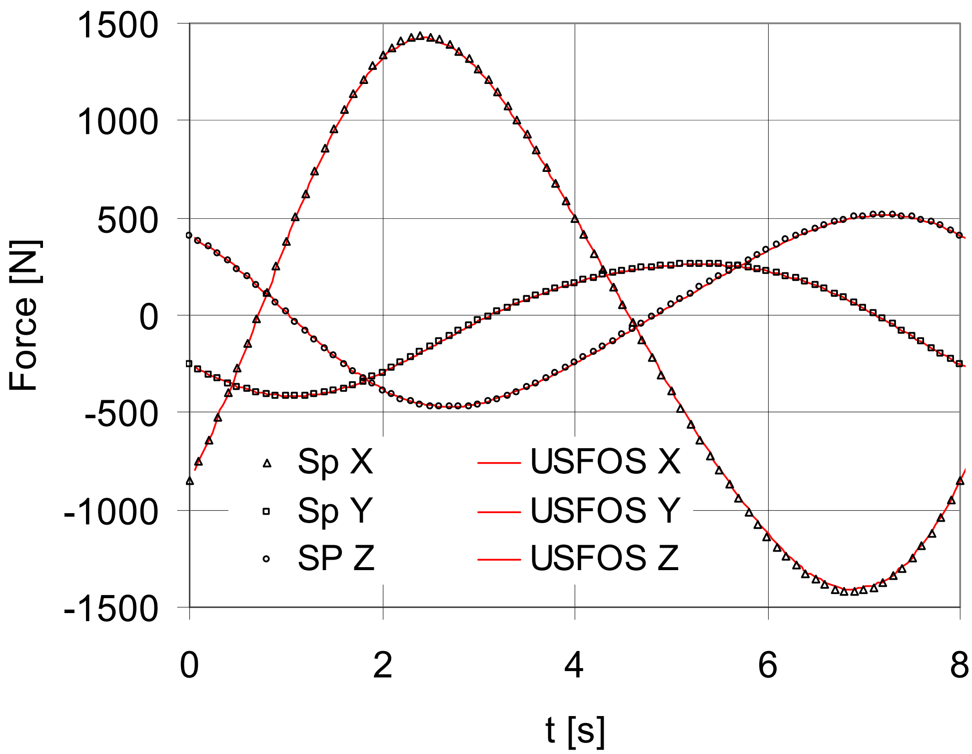


Figure 3.14 Histories of mass force components

3.2.10 Wave and current forces oblique pipe, 20 m depth – Stokes theory

	Period [s]	Height [m]	Theory
	8	5	Stokes
	Depth [m]	Diameter [m]	
	20	0.2	
	C_d	C_m	
	1	0	
	Coord 1 [m]	Coord 2 [m]	
X	0	10	
Y	0	10	
Z	-20	7	
	Reaction SP [N]	Reaction Usfos [N]	Deviation
X	2680.240	2661.940	0.0069
X	-5967.541	-5927.560	0.0067
Y	916.760	907.224	0.0105
Y	-507.017	-502.155	0.0097
Z	2091.478	2079.790	0.0056
Z	-917.873	-912.002	0.0064

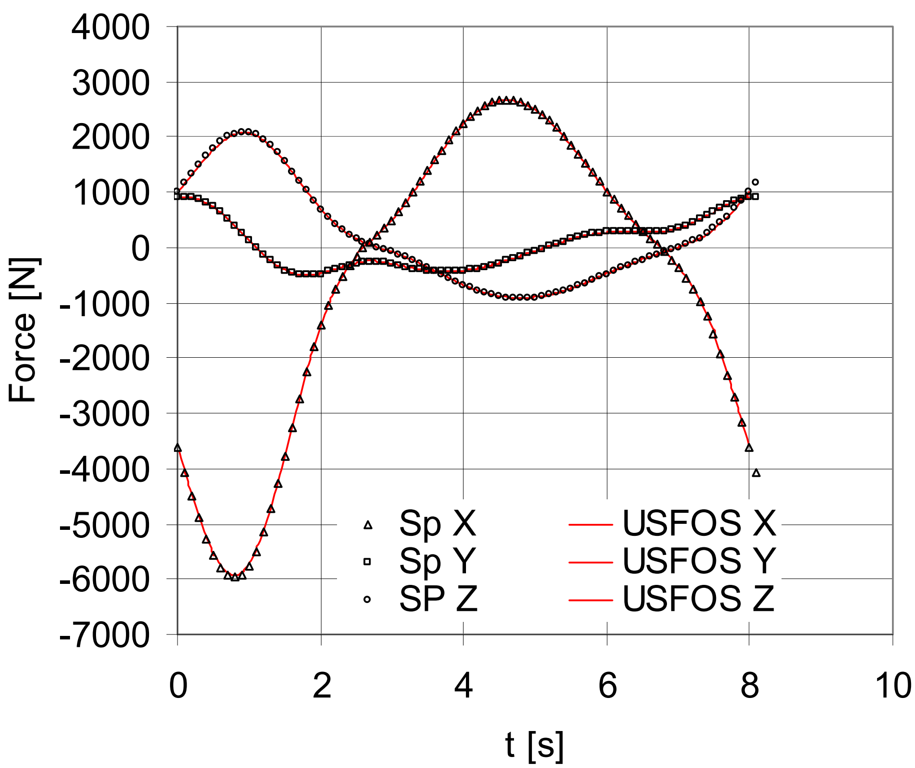


Figure 3.15 Histories of drag force componenets

	Period [s]	**Height [m]**	**Theory**
	8	5	**Stokes**
	Depth [m]	**Diameter [m]**	
	20	0.2	
	C_d	**C_m**	
	0	1	
	Coord 1 [m]	**Coord 2 [m]**	
X	0	10	
Y	0	10	
Z	-20	7	
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X	718.984	713.115	0.0082
X	-708.316	-704.265	0.0058
Y	122.332	121.728	0.0050
Y	-237.223	-236.166	0.0045
Z	263.413	262.106	0.0050
Z	-235.045	-233.253	0.0077

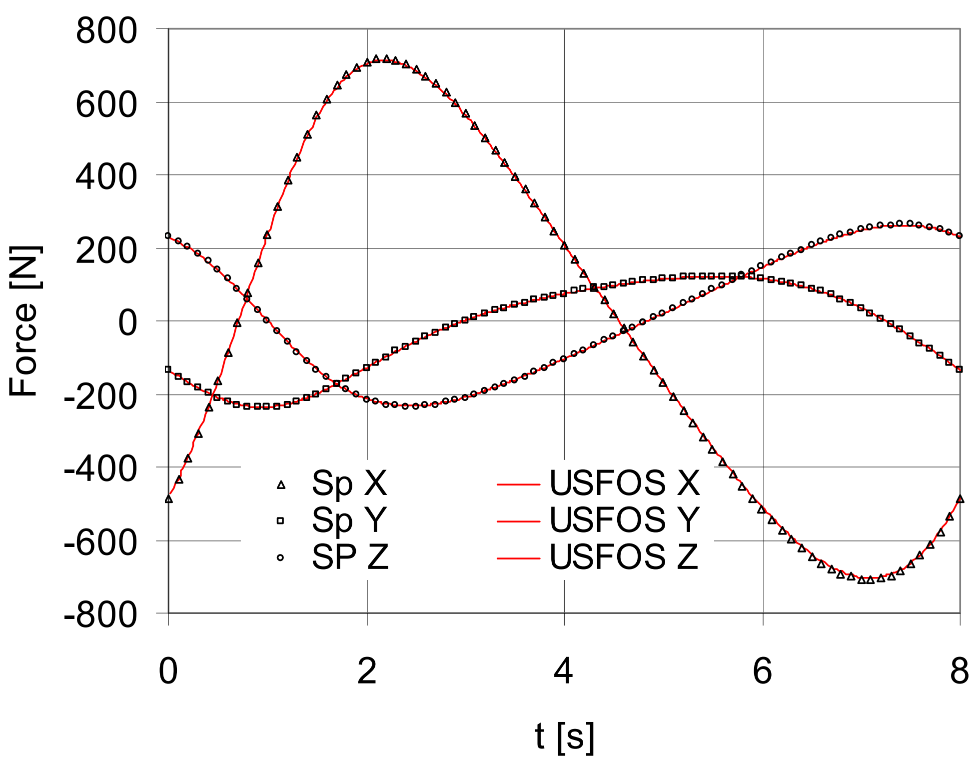


Figure 3.16 Histories of mass force components

3.2.11 Wave forces vertical pipe, 70 m depth – Airy finite depth theory

	Period [s]	**Height [m]**	**Theory**
	15	30	**Finite**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**0**
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	20	0
Y	0	0	**Curr dir**
Z	-70	17	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	84408.782	85542.200	0.0132
X_min	-215841.642	-216760.000	0.0042
Y_max	0.000	0.000	-
Y_min	0.000	0.000	-
Z_max	49618.768	49829.900	0.0042
Z_min	-19404.318	-19664.900	0.0133

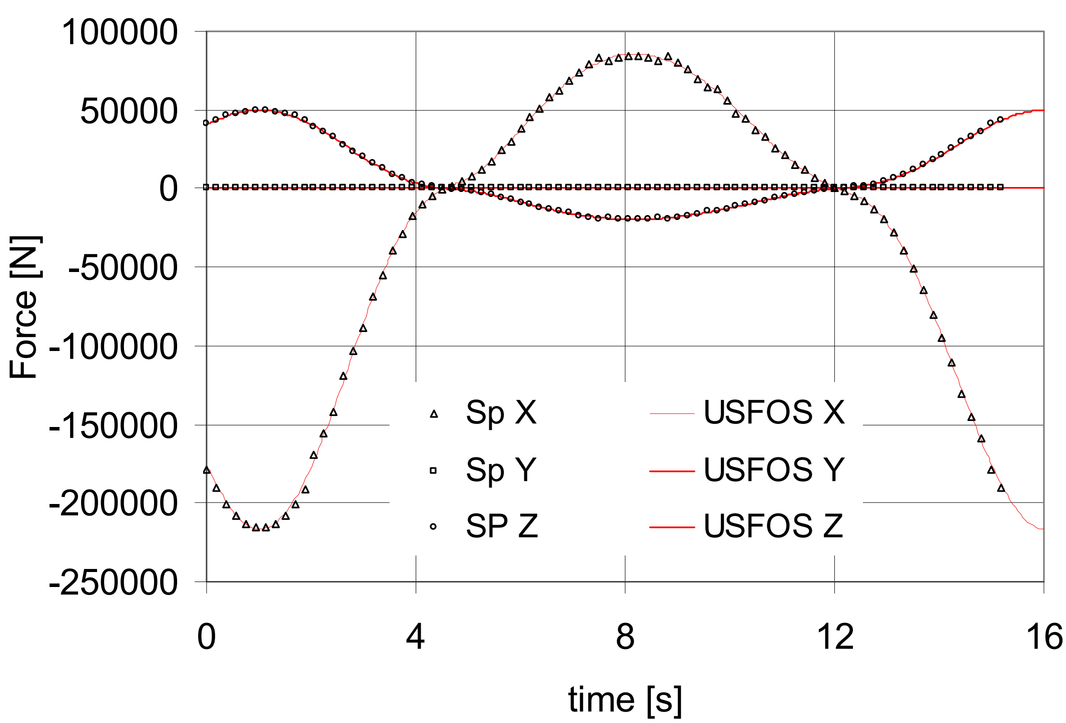


Figure 3.17 Histories of drag force components

	Period [s]	Height [m]	Theory
	15	30	Finite
	Depth [m]	Diameter [m]	Wave dir
	70	0.2	0
	C_d	C_m	
	0	2	
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	20	0
Y	0	0	Curr dir
Z	-70	17	0
	Reaction SP [N]	Reaction Usfos [N]	Deviation
X_max	8714.723	8540.060	0.0205
X_min	-8709.526	-8541.380	0.0197
Y_max	0.000	0.000	-
Y_min	0.000	0.000	-
Z_max	2002.190	1963.540	0.0197
Z_min	-2003.385	-1963.200	0.0205

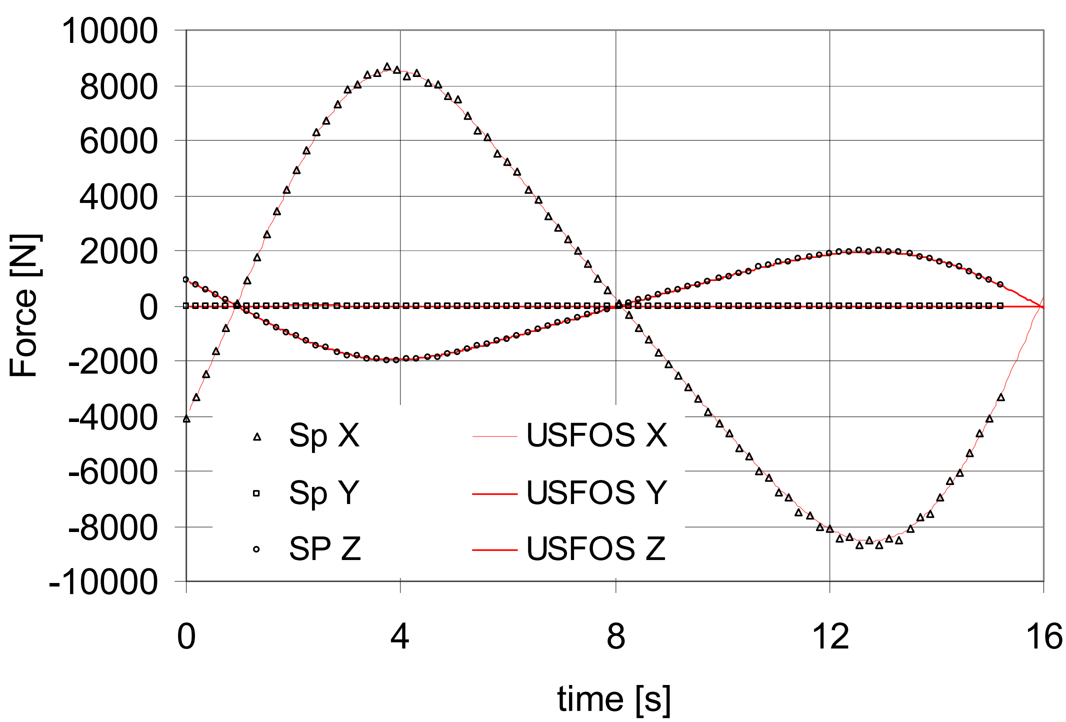


Figure 3.18 Histories of mass force components

3.2.12 Wave forces vertical pipe, 70 m depth – Stokes theory

	Period [s]	**Height [m]**	**Theory**
	15	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**0**
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	20	0
Y	0	0	**Curr dir**
Z	-70	17	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	69270.112	69351.800	0.0012
X_min	-268320.045	-270050.000	0.0064
Y_max	0.000	0.000	-
Y_min	0.000	0.000	-
Z_max	61682.769	62080.600	0.0064
Z_min	-15924.164	-15942.900	0.0012

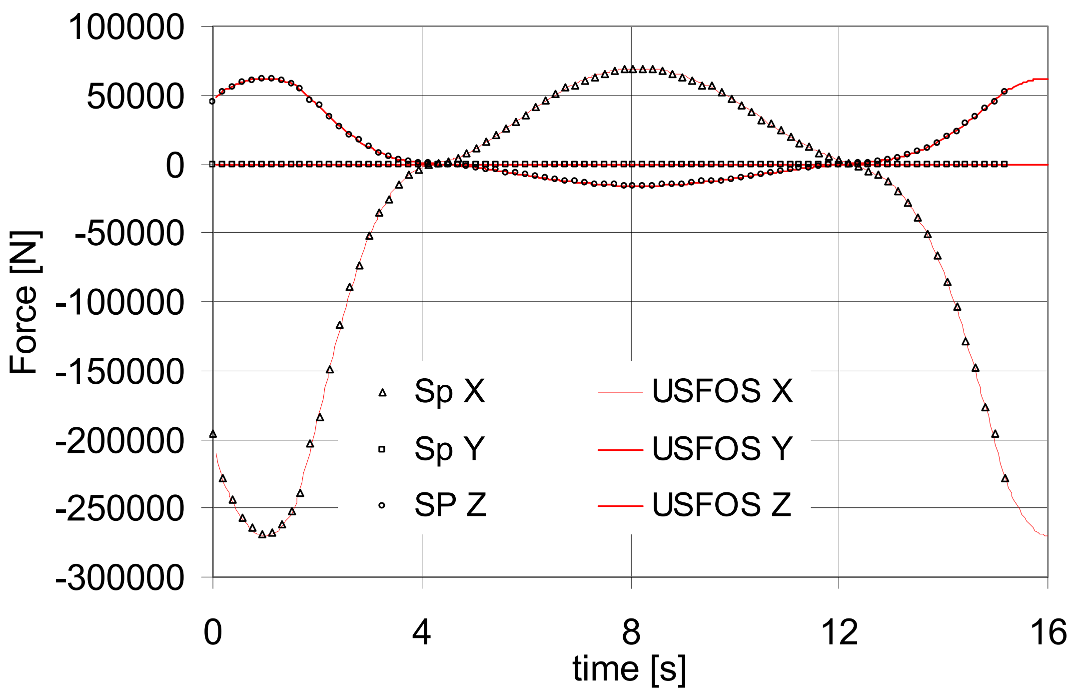


Figure 3.19 Histories of drag force components

	Period [s]	Height [m]	Theory
	15	30	Stokes
	Depth [m]	Diameter [m]	Wave dir
	70	0.2	**0**
	C_d	C_m	
	0	2	
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	20	0
Y	0	0	**Curr dir**
Z	-70	17	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	8643.142	8572.850	0.0082
X_min	-8671.070	-8595.840	0.0088
Y_max	0.000	0.000	-
Y_min	0.000	0.000	-
Z_max	1993.349	1976.060	0.0087
Z_min	-1986.929	-1970.720	0.0082

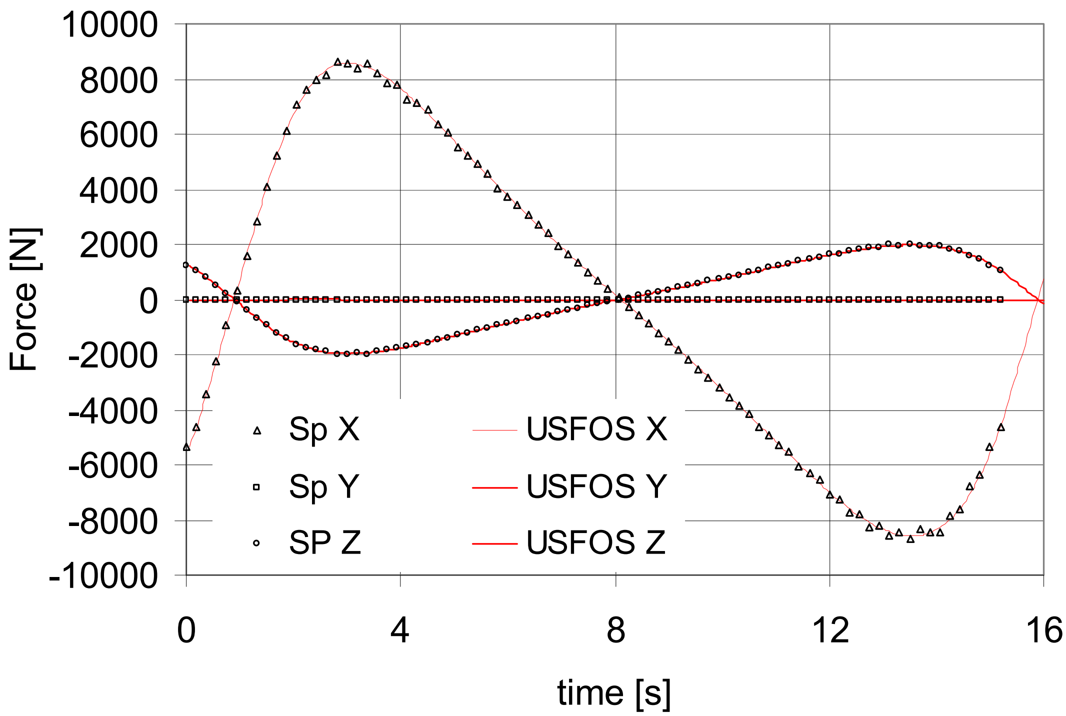


Figure 3.20 Histories of mass force components

3.2.13 Wave forces oblique pipe, 70 m depth – Stokes theory

	Period [s]	**Height [m]**	**Theory**
	15	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**0**
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	30	0
Y	0	30	**Curr dir**
Z	-70	20	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	69931.416	69923.300	0.0001
X_min	-288754.965	-291539.000	0.0095
Y_max	36098.343	36487.700	0.0107
Y_min	-18769.170	-18877.000	0.0057
Z_max	92045.990	92899.500	0.0092
Z_min	-21571.879	-21633.900	0.0029

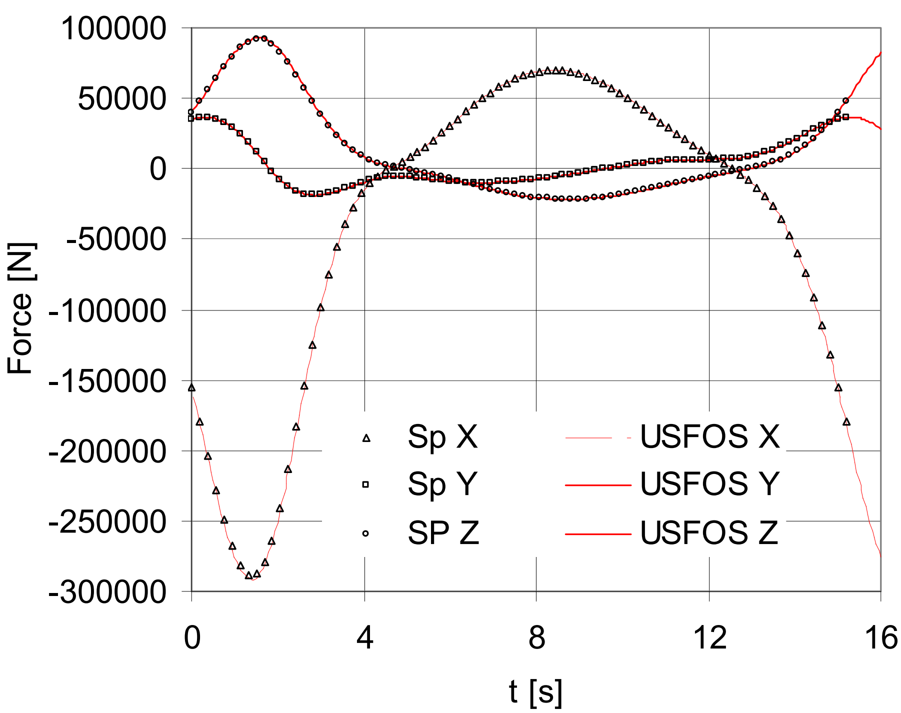


Figure 3.21 Histories of drag force components

	Period [s]	**Height [m]**	**Theory**
	15	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**0**
	C_d	**C_m**	
	0	2	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	30	0
Y	0	30	**Curr dir**
Z	-70	20	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	8895.546	8895.860	0.0000
X_min	-8724.705	-8752.550	0.0032
Y_max	1091.758	1091.260	0.0005
Y_min	-3427.717	-3446.320	0.0054
Z_max	3112.359	3122.900	0.0034
Z_min	-2569.681	-2570.710	0.0004

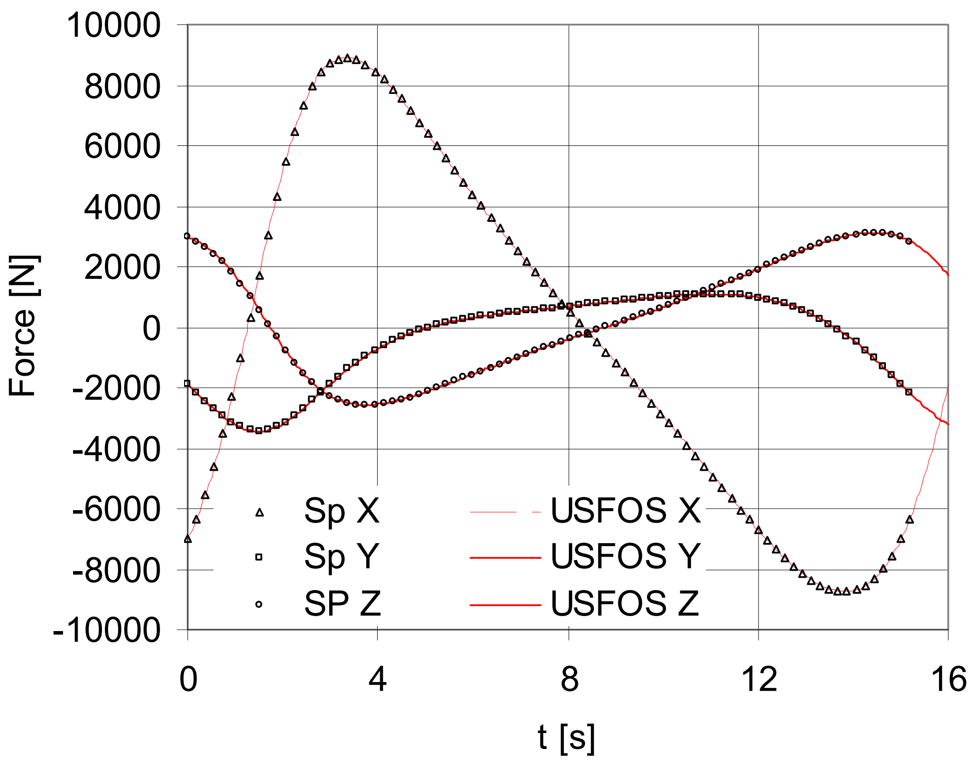


Figure 3.22 Histories of mass force components

3.2.14 Wave forces oblique pipe, 70 m depth, diff. direction – Stokes theory

	Period [s]	**Height [m]**	**Theory**
	15	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**330**
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	30	0
Y	0	30	**Curr dir**
Z	-70	20	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	67159.440	67171.300	0.0002
X_min	-276067.542	-278972.000	0.0104
Y_max	175338.274	176427.000	0.0062
Y_min	-43501.474	-43477.900	0.0005
Z_max	42316.062	42720.800	0.0095
Z_min	-9829.385	-9859.380	0.0030

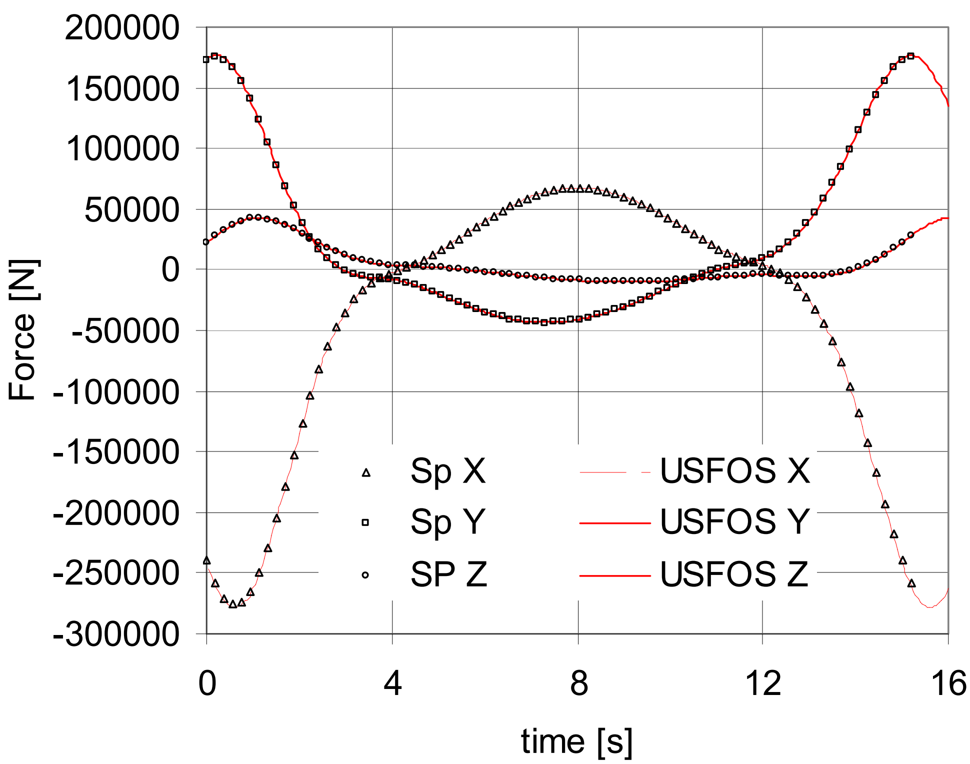


Figure 3.23 Histories of drag force components

	Period [s]	**Height [m]**	**Theory**
	15	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**330**
	C_d	**C_m**	
	0	2	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	30	0
Y	0	30	**Curr dir**
Z	-70	20	0
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	7786.616	7797.800	0.0014
X_min	-8765.111	-8794.150	0.0033
Y_max	4693.902	4690.450	0.0007
Y_min	-6703.430	-6721.900	0.0027
Z_max	2281.966	2295.930	0.0061
Z_min	-1026.651	-1027.110	0.0004

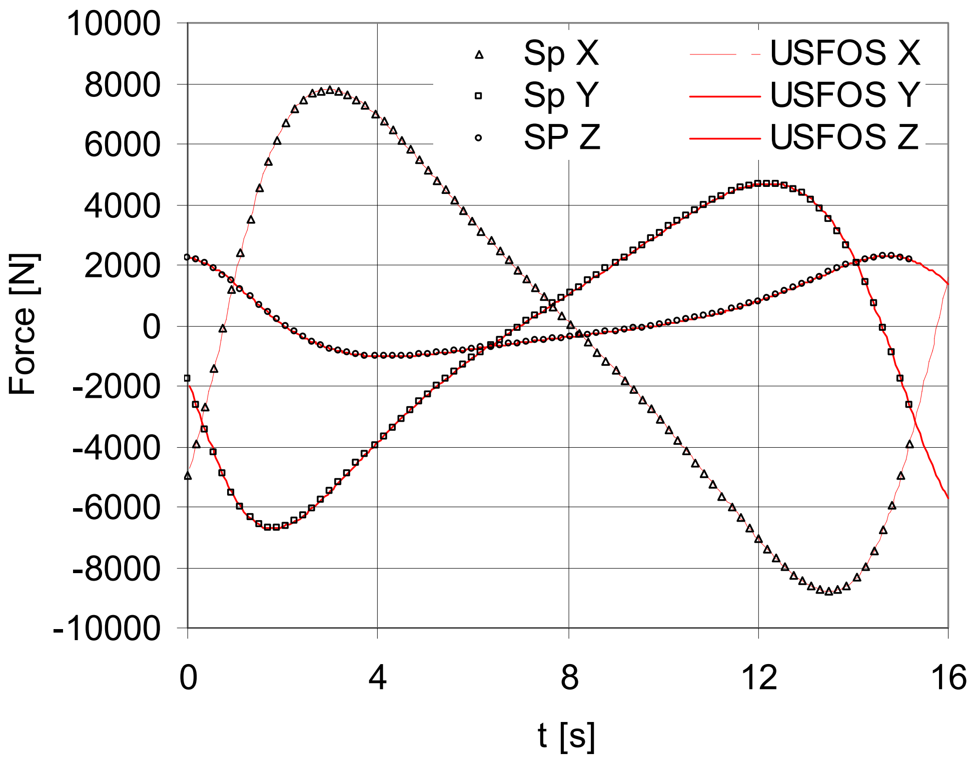


Figure 3.24 Histories of mass force components

3.2.15 Wave forces horizontal pipe, 70 m depth – Airy theory

	Period [s]	**Height [m]**	**Theory**
	15.48	30	**Finite**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	1	**330**
	C_d	**C_m**	Doppler
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	20	1.5
Y	0	30	**Curr dir**
Z	-5	-5	270
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	86838.116	85658.500	0.0138
X_min	-786180.463	-782496.000	0.0047
Y_max	524120.309	521664.000	0.0047
Y_min	-57892.078	-57105.700	0.0138
Z_max	562041.836	559502.000	0.0045
Z_min	-562322.321	-559498.000	0.0050

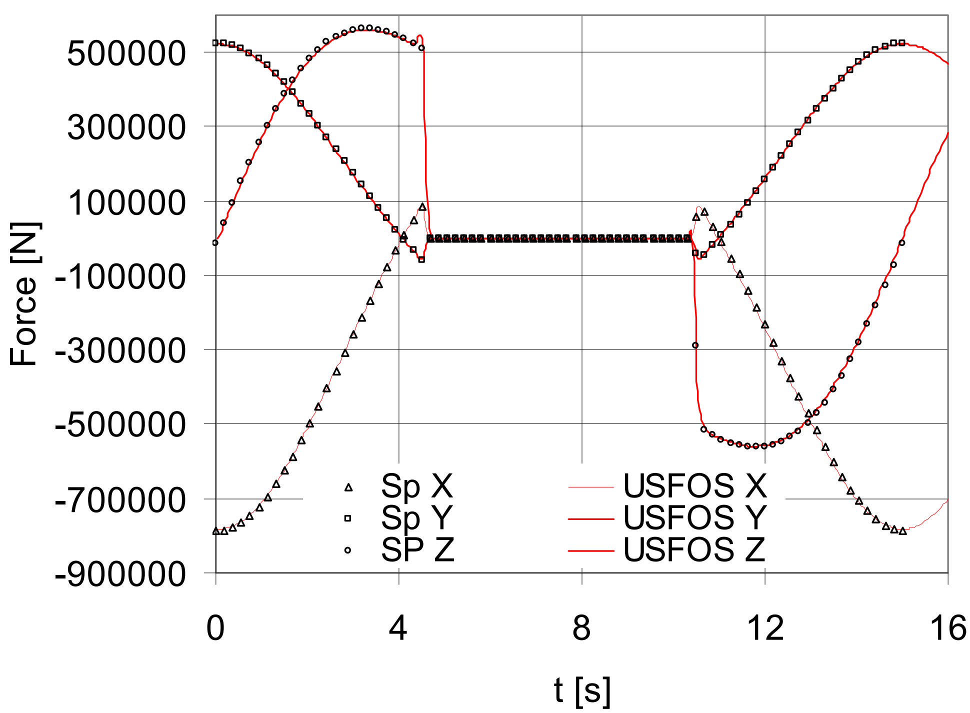


Figure 3.25 Histories of drag force components

	Period [s]	**Height [m]**	**Theory**
	15.48	30	**Finite**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	1	**330**
	C_d	**C_m**	
	0	2	NB doppler
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	20	1.5
Y	0	30	**Curr dir**
Z	-5	-5	270
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	124978.692	125004.000	0.0002
X_min	-124978.693	-125007.000	0.0002
Y_max	83319.129	83335.500	0.0002
Y_min	-83319.128	-83336.900	0.0002
Z_max	128066.478	127460.000	0.0048
Z_min	-36836.672	-35830.800	0.0281

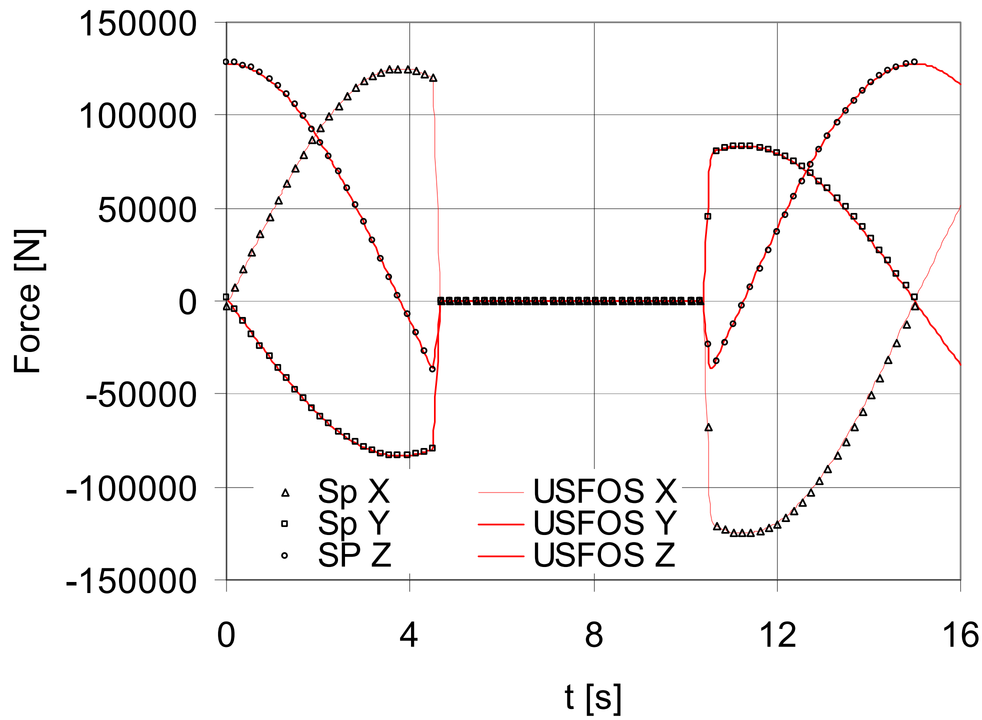


Figure 3.26 Histories of mass force components

3.2.16 Wave forces horizontal pipe, 70 m depth – Stokes theory

	Period [s]	**Height [m]**	**Theory**
	15.48	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	1	**330**
	C_d	**C_m**	NB Doppler
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	20	1.5
Y	0	30	**Curr dir**
Z	-5	-5	270
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	58084.921	49612.400	0.1708
X_min	-825905.087	-819309.000	0.0081
Y_max	550603.391	546206.000	0.0081
Y_min	-38723.281	-33074.900	0.1708
Z_max	505770.775	501198.000	0.0091
Z_min	-506368.805	-501177.000	0.0104

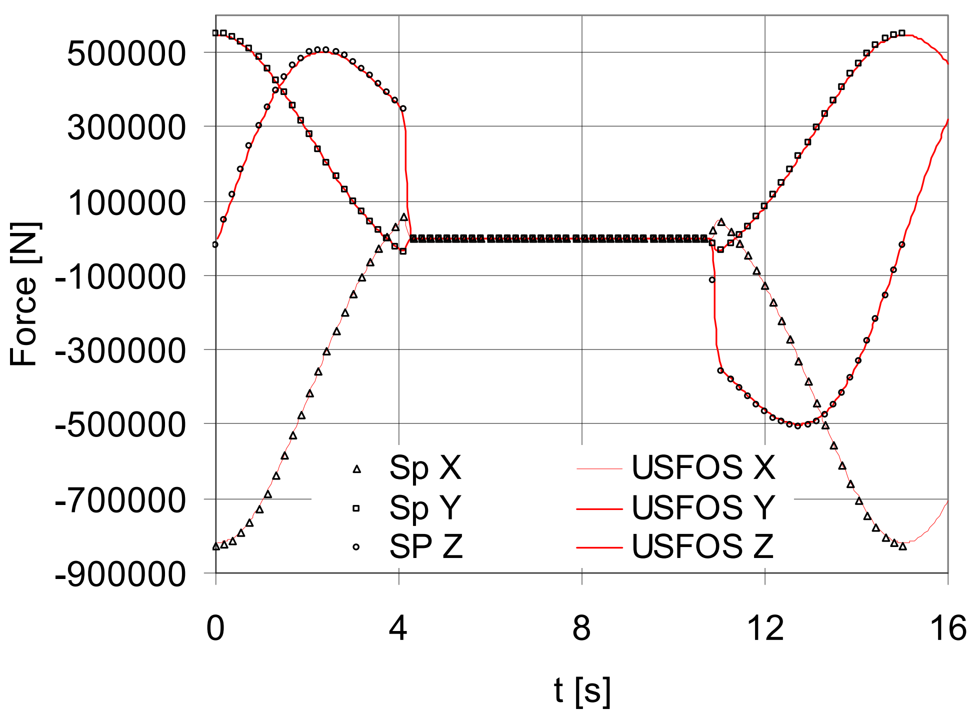


Figure 3.27 Histories of drag force components

3.2.17 Wave and current forces oblique pipe, 70 m depth – Stokes theory

	Period [s]	**Height [m]**	**Theory**
	15	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**330**
	C_d	**C_m**	**NB Teff**
	1	0	15.48
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	30	1.5
Y	0	30	**Curr dir**
Z	-70	20	270
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	60241.319	61653.200	0.0229
X_min	-311909.152	-316263.000	0.0138
Y_max	256922.294	259821.000	0.0112
Y_min	-26979.033	-27699.800	0.0260
Z_max	30036.003	30300.300	0.0087
Z_min	-14585.799	-14434.900	0.0105

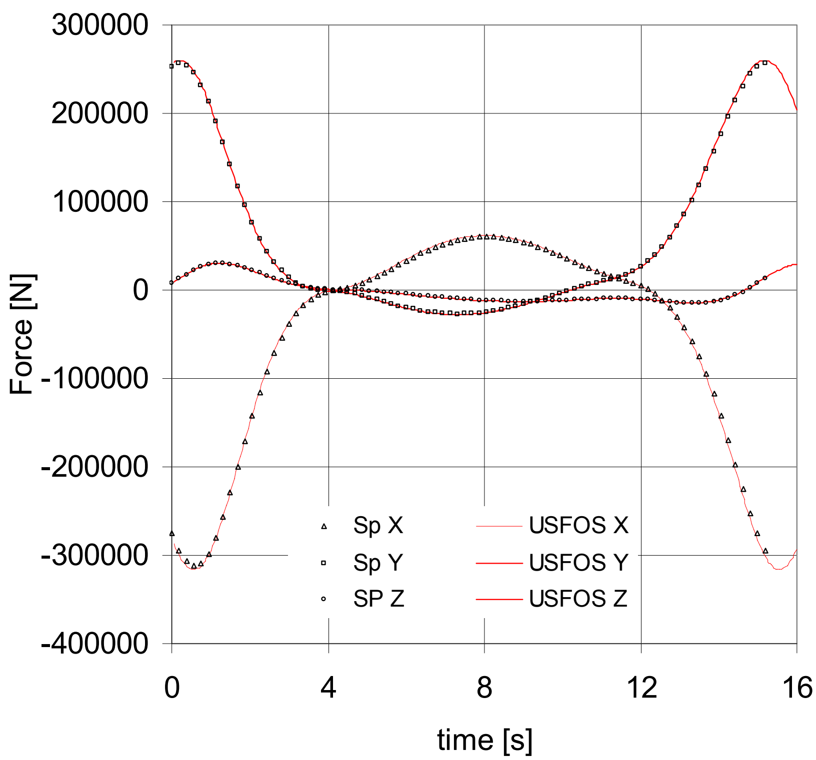


Figure 3.28 Histories of drag force components

	Period [s]	Height [m]	Theory
	15	30	Stokes
	Depth [m]	Diameter [m]	Wave dir
	70	0.2	330
	C_d	C_m	NB Teff
	0	2	15.48
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	30	1.5
Y	0	30	Curr dir
Z	-70	20	270
	Reaction SP [N]	Reaction Usfos [N]	Deviation
X_max	7786.616	7749.640	0.0048
X_min	-8765.111	-8689.150	0.0087
Y_max	4693.902	4665.840	0.0060
Y_min	-6703.430	-6606.830	0.0146
Z_max	2281.966	2229.520	0.0235
Z_min	-1026.651	-1015.950	0.0105

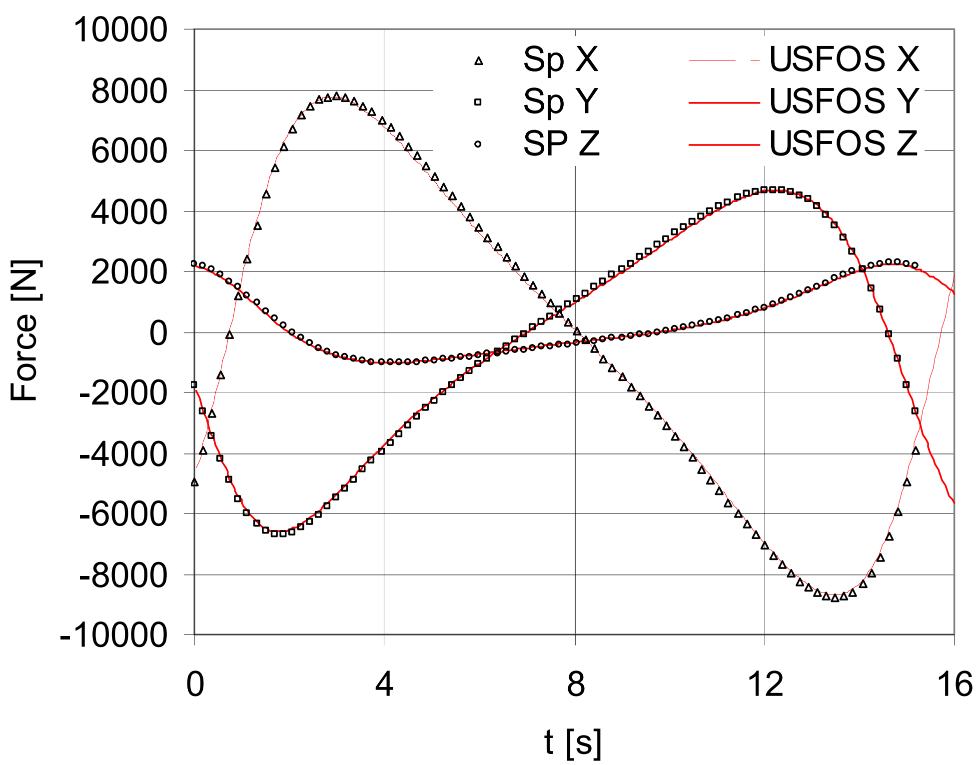


Figure 3.29 Histories of mass force components

3.2.18 Wave and current forces obl. pipe 70 m depth, 10 el. – Stokes theory

			10 EL	**100 EL**
	Period [s]	**Height [m]**	**Theory**	**Theory**
	15	30	**Stokes**	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**	**Wave dir**
	70	0.2	**330**	**330**
	C_d	**C_m**	**NB Teff**	**NB Teff**
	1	0	15.48	15.48
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**	**V_curr**
X	0	30	1.5	1.5
Y	0	30	**Curr dir**	**Curr dir**
Z	-70	20	270	270
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**	**Deviation**
X_max	60241.319	58368.800	0.0321	0.0048
X_min	-311909.152	-310198.000	0.0055	0.0087
Y_max	256922.294	259211.000	0.0088	0.0060
Y_min	-26979.033	-25764.800	0.0471	0.0146
Z_max	30036.003	30353.100	0.0104	0.0235
Z_min	-14585.799	-14968.300	0.0256	0.0105

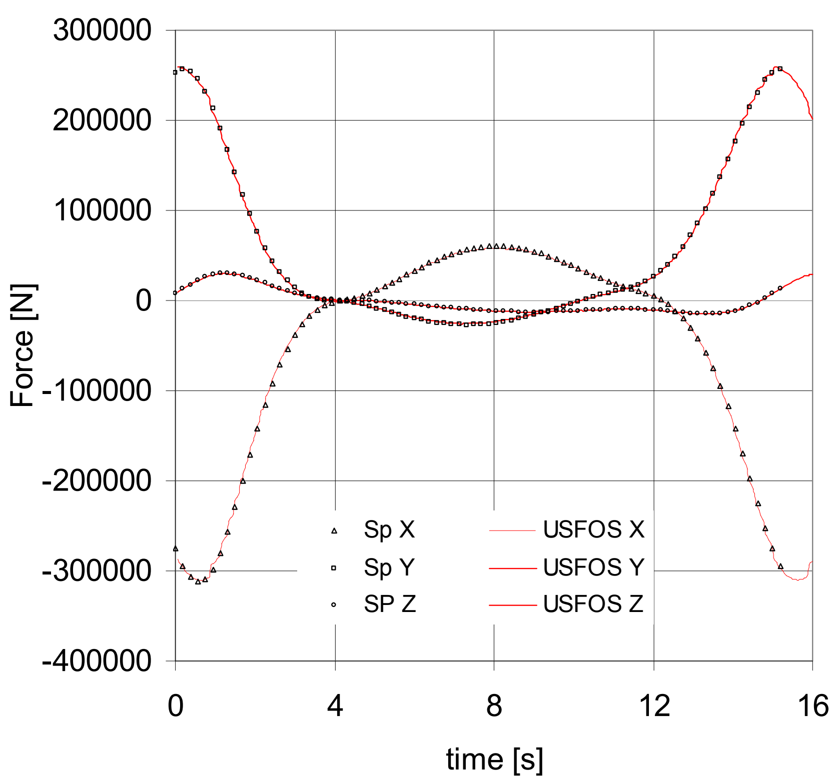


Figure 3.30 Histories of drag force components

3.2.19 Wave and current forces –relative velocity – Airy theory

	Period [s]	**Height [m]**	**Theory**	
	15	30	**Finite**	
	Depth [m]	**Diameter [m]**	**Wave dir**	**V_stru**
	70	0.2	**330**	0.7
	C_d	**C_m**		0.7
	1	0		0
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**	Period [s]
X	0	30	1.5	5
Y	0	30	**Curr dir**	
Z	-70	20	270	
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**	
X_max	78426.228	86832.700	0.0968	
X_min	-223470.653	-232083.000	0.0371	
Y_max	229575.282	258944.000	0.1134	
Y_min	-48842.890	-54157.900	0.0981	
Z_max	37876.147	40491.500	0.0646	
Z_min	-28196.856	-30194.100	0.0661	

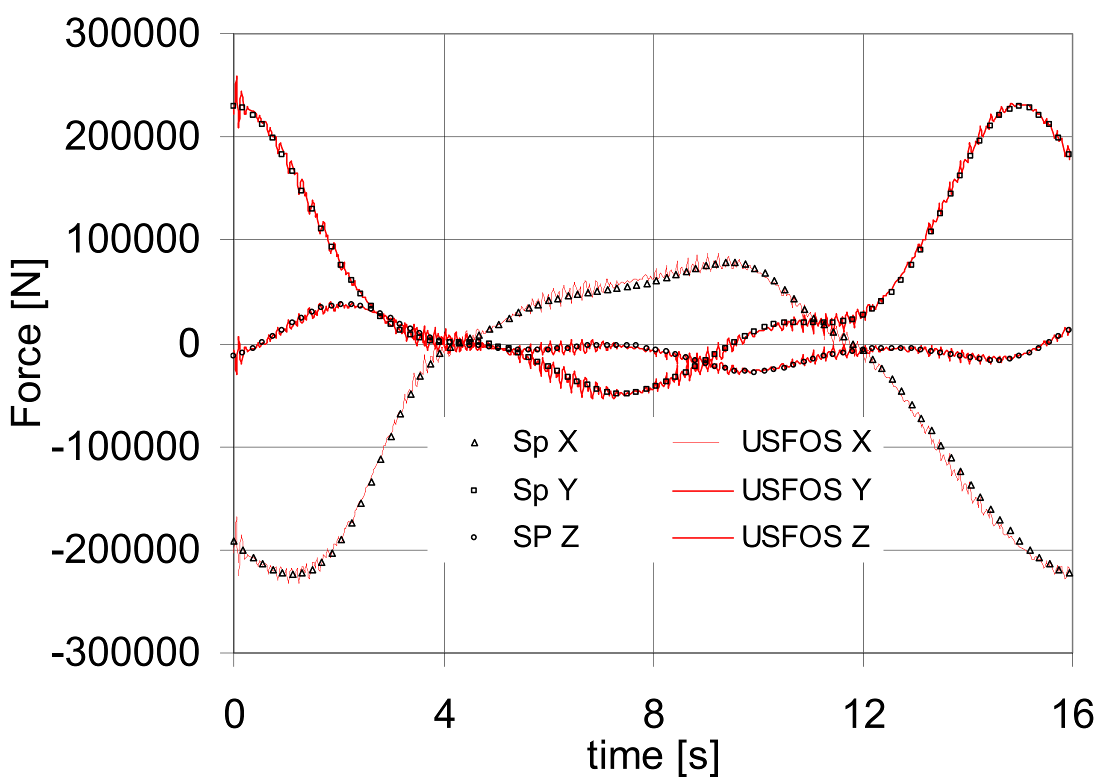


Figure 3.31 Histories of drag force components

3.2.20 Wave and current forces – relative velocity – Stokes theory

	Period [s]	**Height [m]**	**Theory**	
	15	30	**Stokes**	
	Depth [m]	**Diameter [m]**	**Wave dir**	**V_stru**
	70	0.2	**330**	0.7
	C_d	**C_m**		0.7
	1	0		0
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**	Period [s]
X	0	30	1.5	5
Y	0	30	**Curr dir**	
Z	-70	20	270	
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**	
X_max	66682.765	73644.600	0.0945	
X_min	-291773.556	-296639.000	0.0164	
Y_max	292223.326	315818.000	0.0747	
Y_min	-38270.003	-42942.400	0.1088	
Z_max	37924.808	42789.800	0.1137	
Z_min	-24052.753	-28978.800	0.1700	

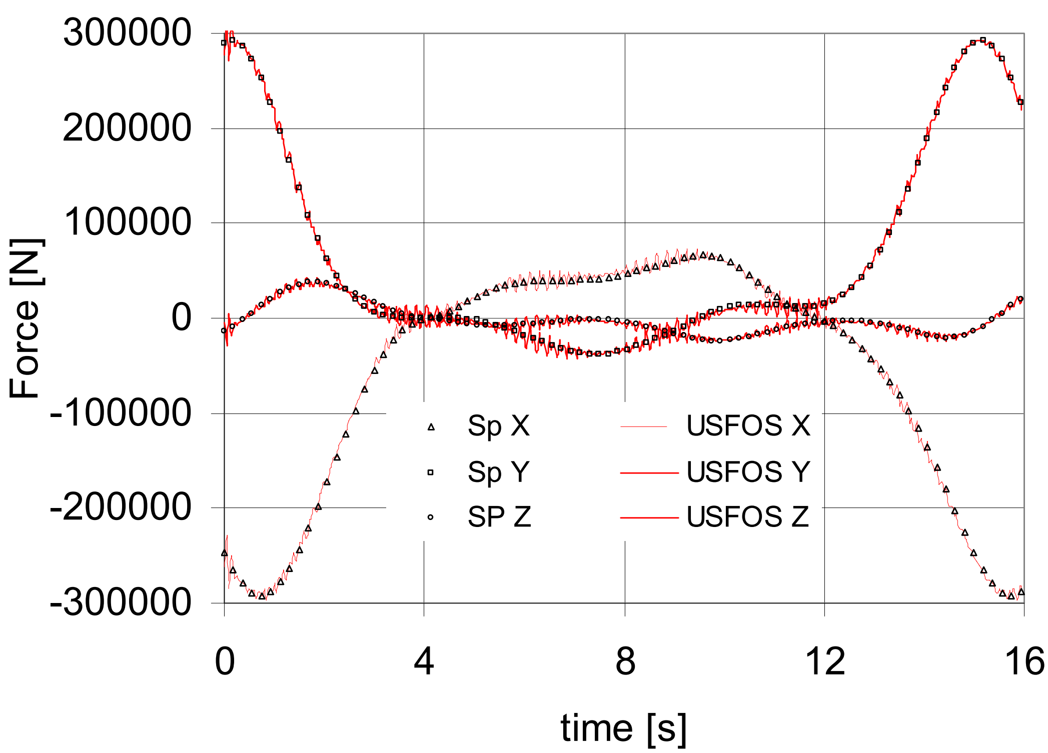


Figure 3.32 Histories of drag force components

3.2.21 Wave and current forces – relative velocity – Dean theory

The effect of relative velocity is checked applying Dean Stream theory. No spreadsheet calculation algorithm has been developed for Dean's theory. The spreadsheet values given are according to Stokes theory: The difference between Stokes and Dean Theory is, however, relatively small for the selected wave height.

	Period [s]	**Height [m]**	**Theory**	**USFOS**
	15	30	**Stokes**	Dean
	Depth [m]	**Diameter [m]**	**Wave dir**	**V_stru**
	70	0.2	**330**	0.7
	C_d	**C_m**		0.7
	1	0		0
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**	Period [s]
X	0	30	1.5	5
Y	0	30	**Curr dir**	
Z	-70	20	270	
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**	
X_max	66682.765	71951.700	0.0732	
X_min	-291773.556	-299467.000	0.0257	
Y_max	292223.326	316815.000	0.0776	
Y_min	-38270.003	-41902.200	0.0867	
Z_max	37924.808	41353.700	0.0829	
Z_min	-24052.753	-29330.300	0.1799	

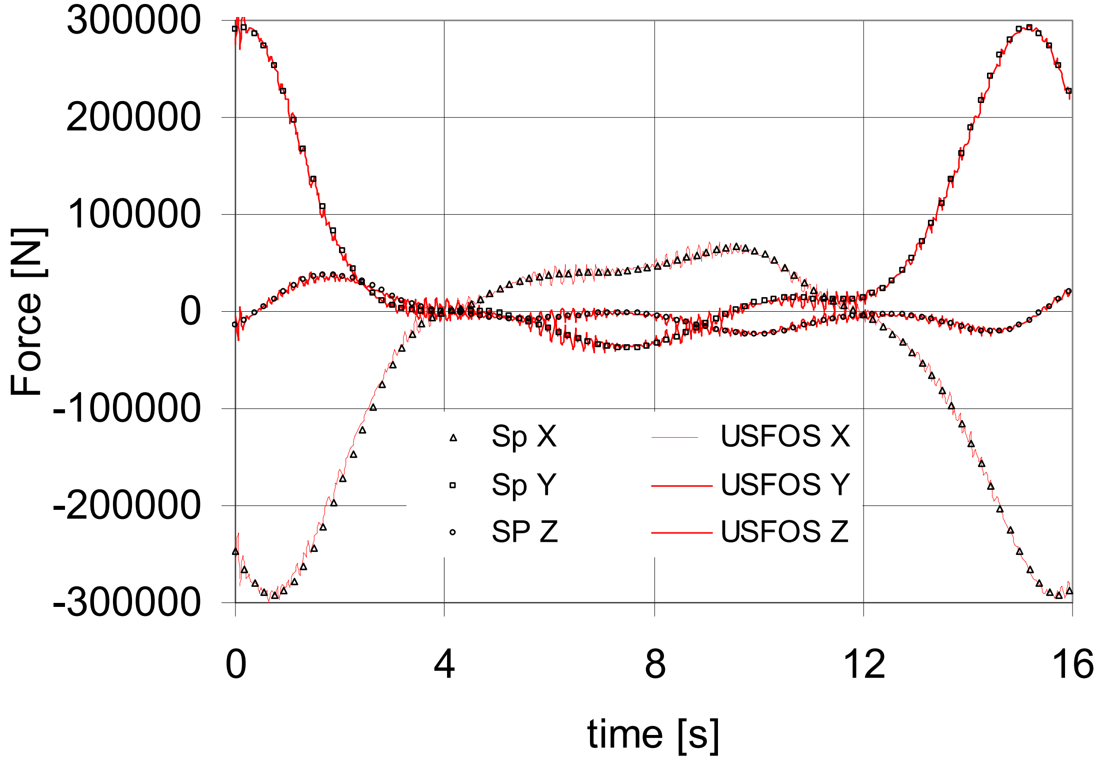


Figure 3.33 Histories of drag force components

3.3 Depth profiles

3.3.1 Drag and mass coefficients

The drag coefficient is assumed to vary linearly with depth with $C_D = 1.0$ at sea floor and $C_D = 2.0$ at sea surface. The mass coefficient is assumed to vary linearly with depth with $C_M = 2.0$ at sea floor and $C_M = 3.0$ at sea surface.

		Period [s]	**Height [m]**	**Theory**
		16	30	**Stokes**
		Depth [m]	**Diameter [m]**	**Wave dir**
		70	0.2	**0**
Linear		**C_d**	**C_m**	
variation		2	0	
		Coord 1 [m]	**Coord 2 [m]**	**V_curr**
	X	0	30	0
	Y	0	30	**Curr dir**
	Z	-70	20	270
		Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
	X_max	108427.907	107891.000	0.0050
	X_min	-560036.819	-559184.000	0.0015
	Y_max	69899.479	69834.200	0.0009
	Y_min	-33981.388	-34009.300	0.0008
	Z_max	178366.600	177760.000	0.0034
	Z_min	-33454.412	-33392.900	0.0018

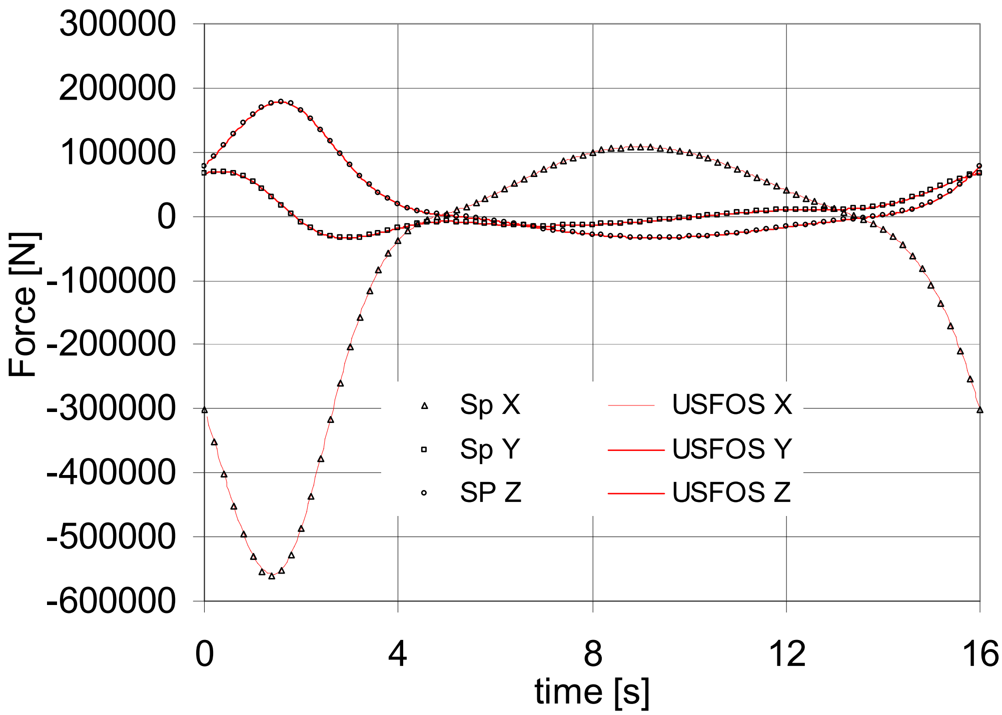


Figure 3.34 Drag force histories

	Period [s]	Height [m]	Theory
	16	30	Stokes
	Depth [m]	Diameter [m]	Wave dir
	70	0.2	0
	C_d	C_m	Linearly
	0	3	Varying
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	30	0
Y	0	30	Curr dir
Z	-70	20	270
	Reaction SP [N]	Reaction Usfos [N]	Deviation
X_max	11529.320	11476.200	0.0046
X_min	-11599.784	-11582.800	0.0015
Y_max	1345.024	1340.390	0.0035
Y_min	-4632.087	-4637.190	0.0011
Z_max	4211.518	4208.120	0.0008
Z_min	-3302.232	-3292.600	0.0029

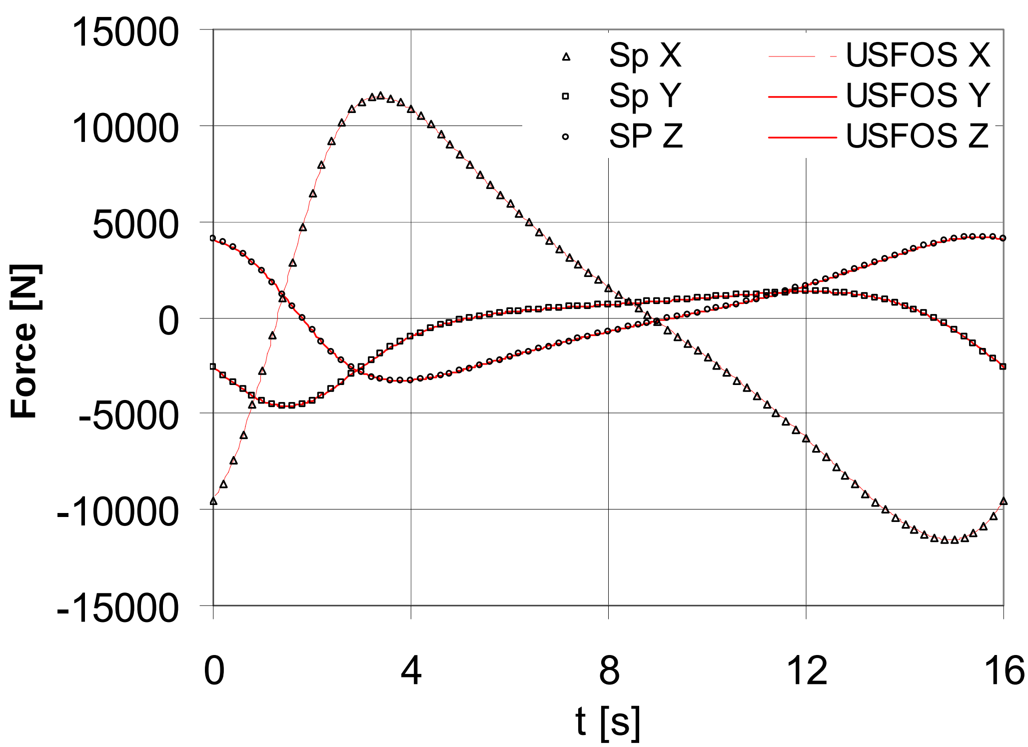


Figure 3.35 Mass force histories

3.3.2 Marine growth

Marine growth is assumed to vary linearly with vertical coordinate. The additional thickness is 0.0 m at sea floor, increasing to 0.05 m at sea surface.

	Period [s]	**Height [m]**	**Theory**
	16	30	**Stokes**
	Depth [m]	**Diameter [m]**	**Wave dir**
	70	0.2	**0**
	C_d	**C_m**	
	1	0	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	30	0
Y	0	30	**Curr dir**
Z	-70	20	270
	Reaction SP [N]	**Reaction Usfos [N]**	**Deviation**
X_max	90828.708	90358.900	0.0052
X_min	-428069.207	-427115.000	0.0022
Y_max	53595.479	53484.900	0.0021
Y_min	-26008.956	-26004.500	0.0002
Z_max	136186.434	135754.000	0.0032
Z_min	-27965.818	-27896.700	0.0025

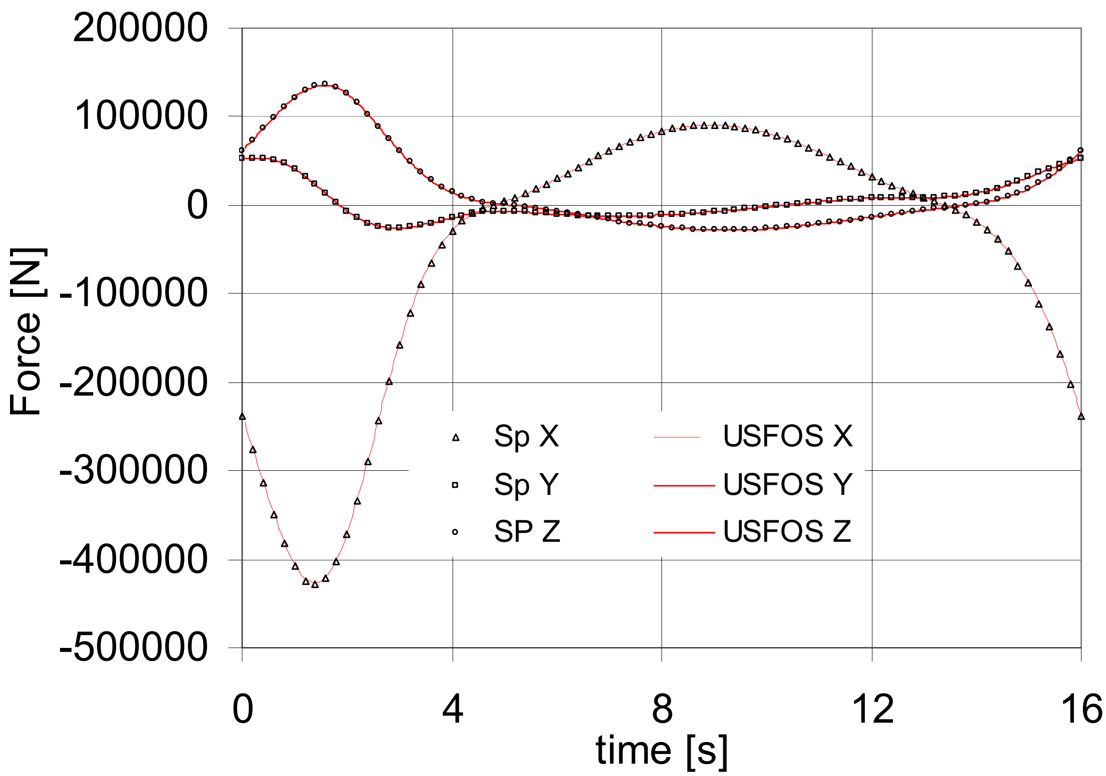


Figure 3.36 Drag force histories

3.4 Buoyancy and dynamic pressure versus Morrison's mass term

3.4.1 Pipe piercing sea surface

The influence of suing direct integration of static and dynamic pressure versus use of Morrison's formulation is investigated. The effect of the pipe piercing the sea surface is also studied. Three different formulations are used:

- Direct integration of hydrostatic and hydrodynamic pressure over the wetted surface
- Morrison's formula with $C_D = 0$, $C_M = 1.0$.
- "Archimedes" buoyancy force, i.e. the force of displaced water, which is equivalent to integration of hydrostatic pressure, only.

When hydrodynamic pressure (BUOYFORM PANEL) is specified the static and hydrodynamic pressure is integrated over the wetted surface. Integration of the dynamic pressure over the cylinder surface (except end caps) should be equivalent to use of Morrison's equation with the mass term with $C_M = 1$. The mass coefficient in Morrison's equation is reduced by 1.0 to account for the hydrodynamic pressure integration. However, end cap effect will be included automatically.

The case study is a pipe located horizontally 5m below sea surface. It is subjected to 30 m high waves. Airy theory for 70 m depth is investigated.

For some period of the wave cycle the pipe is partly or fully out of the water. Fully out of water the Z-reaction equals pipe weight of $7.8 \cdot 10^5$ N, as shown in Figure 3.37.

During wave crest the pipe is fully immersed. The hydrostatic (Archimedes) buoyancy force is $4.1 \cdot 10^5$ N and the corresponding reaction is $3.7 \cdot 10^5$ N. This is calculated correctly.

During wave crest the dynamic pressure is positive, but reduces with depth. The resultant effect is to produce less buoyancy in the vertical direction. This effect is captured correctly, both by direct integration of dynamic pressure and use of Morrison's equation. The reduced buoyancy effect is larger when Morrison's equation is used. This is explained by the fact that Morrison's equation is equivalent to using extrapolated Airy theory while the pressure integration is based in stretched Airy (Wheeler) theory. In the latter case the depth function is smaller and gives smaller dynamic pressure.

When the dynamic pressure is integrated the Y-reaction vanishes correctly for pipe with capped ends. Using Morrison's equation the pressures on the end cap is not included and a resultant, varying Y-reaction is produced.

The X-reactions are similar during wave crests, but deviate significantly when the pipe is about to pierce the sea surface. The X-reaction is larger when dynamic pressure is integrated. This is primarily caused by the X-component of the resulting end cap forces. Because of the phase lag of the dynamic pressure at the two ends, the magnitude of the dynamic pressure (which is negative) is significant at the leading end, which starts piercing the sea surface, while it is almost vanishing at the trailing, submerged end. This produces an additional reaction force.

	Period [s]	**Height [m]**	**Theory**
	16	30	**Airy**
	Depth [m]	**Diameter x thickness [m]**	**Wave dir**
	70	1.2 x 0.08	**0**
	C_d	**C_m**	
	0	1	
	Coord 1 [m]	**Coord 2 [m]**	**V_curr**
X	0	20	-
Y	0	30	**Curr dir**
Z	-5	-5	-

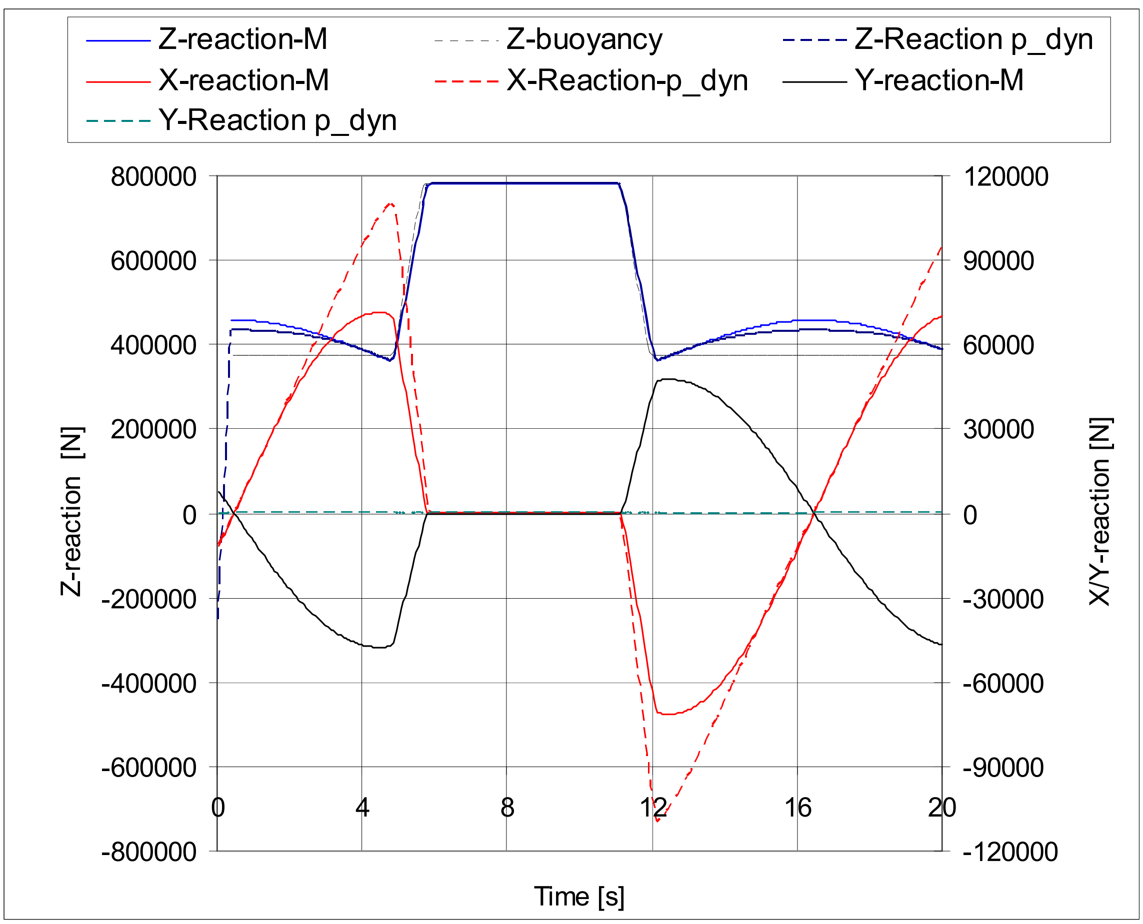


Figure 3.37 Reaction force histories- Airy finite depth theory

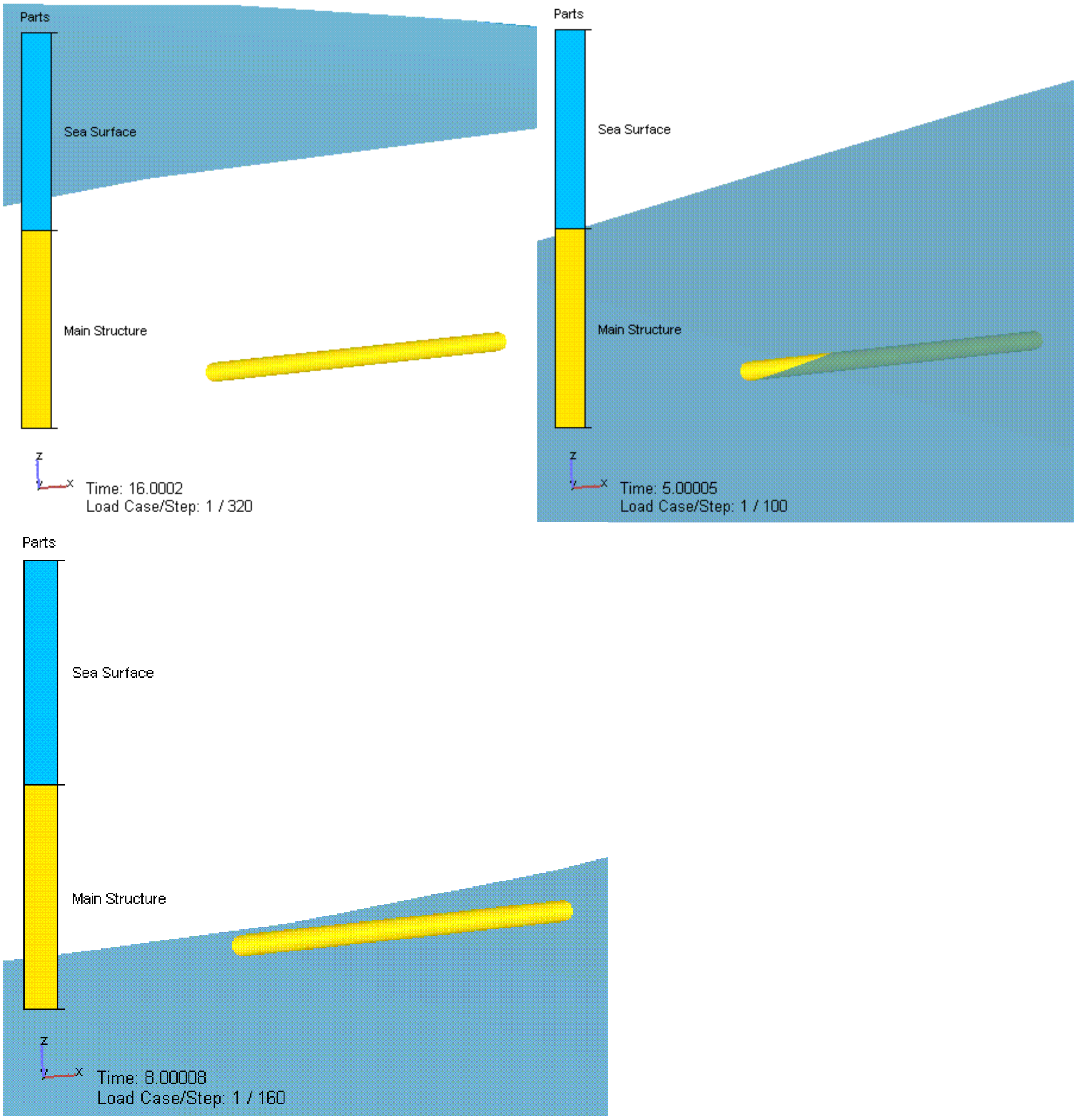


Figure 3.38 Pipe position relative to wave: Fully immersed in wave crest at 0 s (16s), piercing sea surface after 5 s and fully out of water after 8 s.

3.4.2 Fully submerged pipe

This example is identical to the previous one, except that the pipe is located 17 m below mean water level so that is submerged also in the wave trough.

The results are plotted in Figure 3.39. The explanation of the curves is analogous to the ones in the previous example. It is especially noticed that in the wave trough, the dynamic pressure is larger according to stretched Airy theory, giving a larger buoyancy effect (smaller reaction), and the opposite of the situation in the wave crest.

	Period [s]	Height [m]	Theory
	16	30	Airy
	Depth [m]	Diameter x thickness [m]	Wave dir
	70	1.2 x 0.08	0
	C_d	C_m	
	0	10	
	Coord 1 [m]	Coord 2 [m]	V_curr
X	0	20	-
Y	0	30	Curr dir
Z	-17	-17	-

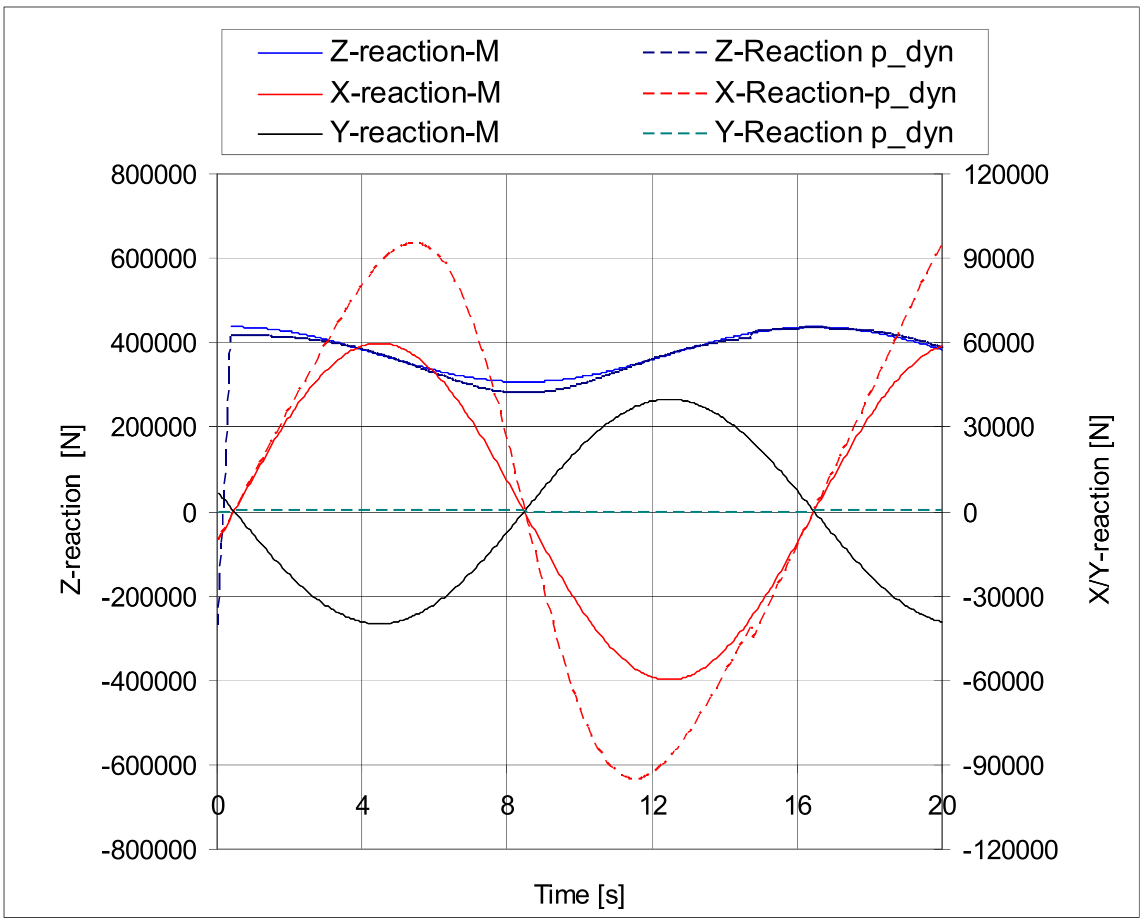


Figure 3.39 Reaction force histories – 70 m depth

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