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Collapse Analysis of Framed Offshore Structures

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ABSTRACT

The paper presents new approaches for nonlinear system analysis of trusswork platforms. The main idea behind the technique is to minimize the cost of the nonlinear analysis by reducing the number of parameters and the size of element model.

A brief description is given on the theoretical basis of the computer program including the formulation of incremental stiffness with modifications for plastic hinges. Special emphasis is given to the choice of adequate interpolation functions and to the evaluation of nonlinear element characteristics.

The solution strategy follows the conventional incrementation procedure with special algorithms for handling stability problems and load reversal. The numerical procedure is demonstrated on buckling analysis of simple columns and frames.

Comparison with experiments and alternative numerical computations proves the accuracy and efficiency of the procedure. Practical design examples are given on progressive collapse analysis of a jacket structure, a deep water tripod typer of platform and a module support frame.

INTRODUCTION

The implementation of accidental loads as a design limit state in offshore rules (DnV,1981) has strengthened the need for rational tools for such calculations. It is of interest to know the amount of damage as well as the residual strength in damaged condition.

Much effort has been given to the development of refined nonlinear finite element programs during the last decade and several program systems are available for special purpose analysis. However, for the designer the step from conventional ultimate load design to the use of advanced nonlinear finite element or finite difference programs is still difficult. Special problems arises during the choice of numerical model, e.g. in the subdivision of a bracing element into an adequate number of finite elements so as to pick up possible local element buckling.

The aim of the present work is to create a more design oriented numerical tool in the sense that only one numerical element is used per physical element. Further, the plastic behaviour is described by interaction formulas for cross-sectional capacity which also are well defined characteristics in conventional design.

The basic idea behind this socalled Idealized Structural Unit Method was described (Ueda and Rashed, 1974) for ultimate strength analysis of transverse frames in ship structures. The procedure was extended for ultimate strength analysis of tubular frame structures (Rashed, 1980). Further development and adoption to progressive collapse analysis of mobile offshore platforms was presented (Aanhold, 1983) including the modelling of damaged structural members. The dented region was replaced by an eccentric circular cylinder with equivalent cross section properties.

The extension of the technique (Søreide, 1986) includes refinement of the formulation for large deflections as well as the solution strategy. Due to the coarse element mesh nonlinear geometric behaviour on element level is modelled allowing moderate deflections between nodal points. The basic continuum theory behind the method is also revised so that the stiffness is derived from the incremental form of the virtual work principle. Interesting developments are now going on to incorporate concrete elements of different cross sections.

References and illustrations at end of paper.

THEORETICAL BASIS

The formulation behind the program is valid for large displacements but restricted to small strains. Figure 1 shows the local set of axis for a beam element with the deflected shape dotted. The total displacement of a point P is decomposed into axial displacement u(x) and lateral deflection v(x) (and w(x) in three dimensions). The expression for strain is subsequently established in local element xy-system and the element stiffness is also derived in local system.

$$\varepsilon_x = u,_x + \frac{1}{2}u,_x^2 + \frac{1}{2}v,_x^2 + \frac{1}{2}w,_x^2 \qquad (1)$$

which for moderate rotations on element level is simplified into

$$\varepsilon_x = u,_x + \frac{1}{2}v,_x^2 + \frac{1}{2}w,_x^2 \qquad (2)$$

The internal strain energy for the elastic beam element reads

$$U = \frac{1}{2}\int_0^\ell EA(u,_x + \frac{1}{2}v,_x^2 + \frac{1}{2}w,_x^2)^2dx + \frac{1}{2}\int_0^\ell (EI_z v,_{xx}^2 + EI_y w,_{xx}^2)dx \qquad (3)$$

where the first integral comes from axial straining and the last integral represents bending. Torsion is not included in the variational formulation but is added directly into the element stiffness matrix.

The potential of external loads is written as

$$H = -(F_i u_i + \int_0^\ell q_x \cdot udx + \int_0^\ell q_y \cdot vdx + \int_0^\ell q_z \cdot wdx) \qquad (4)$$

The total potential for an elastic element is now

$$\pi = U + H \qquad (5)$$

The first variation of internal strain energy comes out of equation 3 as

$$\begin{aligned}\delta U = &\int_0^\ell EAu,_x \cdot \delta u,_x dx \\ &+ \int_0^\ell EI_z(v,_{xx} \cdot \delta v,_{xx} - \frac{N}{EI_z} v,_x \cdot \delta v,_x)dx \\ &+ \int_0^\ell EI_y(w,_{xx} \cdot \delta w,_{xx} - \frac{N}{EI_y} w,_x \cdot \delta w,_x)dx \\ &- \int_0^\ell (N+EAu,_x)\delta u,_x dx \end{aligned} \qquad (6)$$

The first term in equation 6 is the linear contribution from axial strain. The two next integrals represents bending deformation including magnification due to axial compression force N. The corresponding stiffness matrix are represented by the Livesley functions (Livesley, 1975). The last integral comes from the nonlinear axial strain contribution from lateral deflections and gives a correction to the equilibrium axial loads. Equation 6 is the basis for calculating internal equilibrium forces to be compared with external loads during equilibrium correction.

Denoting by Δ the increment between two close configurations the variation of increment in strain energy reads

$$\begin{aligned}\delta\Delta U = &\int_0^\ell EA\Delta u,_x \delta u,_x dx \\ &+ \int_0^\ell EI_z(\Delta v,_{xx}\delta v,_{xx} - \frac{N}{EI_z} \Delta v,_x \delta v,_x) \\ &+ \int_0^\ell EI_y(\Delta w,_{xx}\delta w,_{xx} - \frac{N}{EI_y} \Delta w,_x \delta w,_x) \\ &+ \int_0^\ell EA(\Delta u,_x v,_x \delta v,_x + \Delta v,_x v,_x \delta u,_x)dx \\ &+ \int_0^\ell EA(\Delta u,_x w,_x \delta w,_x + \Delta w,_x w,_x \delta u,_x)dx \\ &+ \int_0^\ell EA\Delta v,_x v,_x^2 \delta v,_x dx \\ &+ \int_0^\ell EA\Delta w,_x w,_x^2 \delta w,_x dx \\ &+ \int_0^\ell EA(\Delta v,_x v,_x w,_x \delta w,_x + \Delta w,_x w,_x v,_x \delta v,_x)dx \\ &+ \text{higher order terms} \end{aligned} \qquad (7)$$

The incremental stiffness is obtained by interpolating element displacements. It is seen from equation 7 that this results in a symmetric stiffness matrix.

The interpolation functions used for deflections satisfy the differential equations for a beam with axial force N (positive in compression) and no lateral load. Introducing the notation

$$k^2 = \frac{N}{EI_z} \qquad (8)$$

gives for N in compression the deflection function

$$v(x) = A_1 \cdot \cos kx + A_2 \sin kx + A_3 x + A_4 \qquad (9)$$

where A_{1-4} are generalized coordinates. For N in tension the shape function for v(x) is

$$v(x) = A_1 \ e^{kx} + A_2 e^{-kx} + A_3 \ x + A_4 \qquad (10)$$

Similar expressions are used for w(x). The interpolation of axial displacement u(x) was originally linear. However, with the strain expression of equation 2 self-straining happened to occur in quite many problems since the linear term $u,_x$ does not match the higher order contributions. Interpolation of u(x) is therefore chosen as for v(x) and w(x).

The local element stiffness matrix from the above expressions is subsequently modified for plasticity. The present version of the program considers three alternative locations of plastic hinges for each element, namely at the element ends and at midspan.

The case of plastic hinge at the element ends is considered first. The interaction formula for the cross section may generally be expressed in terms of stress resultants

$$\Gamma(N,Q_y,Q_z,M_x,M_y,M_z) = 0 \qquad (11)$$

where the notation is as follows

N - axial force

Q_y - shear force in local y-direction

Q_z - shear force in local z-direction

M_x - torsion moment

M_y - bending moment about local y-axis

M_z - bending moment about local z-axis

Equation 11 defines the plastic state of stress resultants while elastic situations are characterized by negative Γ.

Elastic and plastic displacements are now separated

$$v = v^e + v^p \qquad (12)$$

and the normality condition introduced for the increment in plastic displacement for the hinge at end no "i"

$$\Delta v_i{}^p = \Delta\lambda_i g_i \qquad (i = 1,2) \qquad (13)$$

$$g_i{}^T = \left[\frac{\partial\Gamma}{\partial N}, \frac{\partial\Gamma}{\partial Q_y}, \frac{\partial\Gamma}{\partial Q_z}, \frac{\partial\Gamma}{\partial M_x}, \frac{\partial\Gamma}{\partial M_y}, \frac{\partial\Gamma}{\partial M_z},\right]_i \qquad (14)$$

The consistency condition, stating that the stress state lies on the interaction surface during plastic deformation of the hinge gets the form for hinge no "i"

$$\Delta\Gamma_i = g_i{}^T \Delta S_i = 0 \qquad (i = 1,2) \qquad (15)$$

where ΔS_i is the vector of incremental stress resultants and node no "i" of the element.

$$\Delta S_i{}^T = [\Delta N, \Delta Q_y, \Delta Q_z, \Delta M_x, \Delta M_y, \Delta M_z]_i \qquad (16)$$

Combining equations 12-16 gives the elasto-plastic incremental stiffness matrix for a beam element with local large deflection effects and modification for plasticity

$$\begin{bmatrix} \Delta S_1 \\ \\ \Delta S_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ & \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \Delta v_1 \\ \\ \Delta v_2 \end{bmatrix} \qquad (17)$$

The set of displacement parameters and stress resultants for a three-dimensional beam element is indicated in Figure 2.

In the case of a plastic hinge at midspan the actual element is divided into two new subelements. The elastic stiffnesses for the two subelements are found with large deflection effects included. A plastic hinge is introduced in one of the two subelements at the intermediate node and the stiffness matrix for this subelement is modified, see Figure 3. Finally the midnode is eliminated by static condensation and the original member 1-2 is again the basic element to further include in the global frame analysis. The process of static condensation is performed at element level and does not imply much computer cost.

The further assemblage of element stiffnesses into the global system stiffness is straightforward. A spatial reference system, X, Y, Z has the following functions:

a. To serve as the reference system for global frame geometry.

b. To be the reference system for global load-displacement relations. Interelement continuity is established through transformation of element stiffnesses into the spatial system before adding.

The choice of the reference systems is illustrated in Figure 4.

The process of developing numerical schemes for solving the nonlinear equations is continuously running as new design examples require special techniques. It is now felt that through a wide range of case studies the solution algorithm has gained a quality that can handle most practical analyses of progressive collapse.

The pure Euler-Cauchy incrementation method is combined with equilibrium corrections as illustrated in Figure 5. A further extension is to use equilibrium iterations at each level of external load.

The introduction of plastic hinges may give abrupt changes in the system behaviour and special care must be taken concerning load increments. A scaling of increment size is applied when plastification occurs so that the failure surface at the new hinge is not exceeded.

NUMERICAL STUDIES

Based on the above theory a computer program USFOS has been developed. The program is aimed at Ultimate Strength analysis of Framed Offshore Structures. Numerical studies are presented for simple beam-columns, planar frames and more complicated spatial frames. Comparison is made with nonlinear finite element analyses and with experiments.

Example 1: Simple beam-column

As a first test of the program the simply supported beam-column in Figure 6 is studied. The slenderness ratio is $\ell/r = 120$. The present program USFOS models the beam-column by one single element. It appears that the load-deformation relationship predicted is close to the results obtained by the nonlinear finite element program FENRIS (Manuals, 1985). The results are virtually identical in the elastic range and the deviation at the peak load is attributed to the fact that linear elastic-perfectly-plastic moment/curvature relationship is assumed in USFOS, so that the effect of gradual plastification of the cross section is not inherent. In the post-buckling range the capacity is underestimated by USFOS since initial out-of-straightness is simulated by lateral load.

It is interesting to observe that the simple Euler-Cauchy incrementation procedure compare well with step-iterative technique except at the peak load. This difference decreases for smaller slendernesses, as normally found in jacket structures. Hence, it is concluded that in practical design satisfactory results can be obtained without the use of equilibrium iterations.

Example 2: Beam with membrane action

The study in Figure 7 is considered to be critical with respect to the displacement shape functions. The deflection of a simply supported beam under lateral uniformly distributed load is a four degree polynomial in the linear range rather than the hyperbolic sine and cosine functions in USFOS. However, the initial stiffness as well as the large deflection behaviour are fairly well predicted as compared with finite element calculations (Søreide, 1977).

Example 3: K-shaped frames

The next verification study concerns experiments with two planar K-shaped frames shown in Figure 8. The transverse beam is supported by two diagonal bracing elements and loaded by a hydraulic jack at the joint. Figure 9 gives the test results for vertical load versus vertical displacement of the joint for the case of diagonal dimensions 101.7 · 3.30 W.T. (mm).

The numerical analysis is based upon lower yield stress 335 MPa in the diagonals where buckling is predominant and maximum yield stress 414 MPa for the transverse beam to account for strain hardening in the large deflection range. It is seen that USFOS overestimates slightly the ultimate load whereas in the post-ultimate range the capacity is underestimated. Taking into consideration the sensitivity of the test case and the uncertainties in determination of yield stress the deviation is tolerable.

Example 4: 3-D frames

Two experiments on 3-D frames are to be performed. The frames in Figure 10 are designed so as to resemble a subsystem of a four-legged jacket and to exhibit a collapse behaviour including several load redistribution capabilities. A quasistatic load is applied paralell to element EF at joint E.

The load-deformation curves predicted are plotted in Figure 11 and shows a significant reserve strength. The later tests will supplement these numerical studies.

Example 5: Jacket structure

Figure 12 shows the element model of an eight pile jacket typical for installations at about 70 m water depth in the North Sea. All framings are X-braced and the members have been dimensioned according to the API-RP2A code. The piles are included in the computer model without any consideration of foundation capacity. The deck is represented by beam elements with typical stiffness of deck trusses. Collapse of the deck is not modelled in the present study.

Loading consists of functional and environmental loads with design waves of a return period equal to 100 years. Three wave directions are considered, namely longitudinal, 45^{0} inclined and transverse.

As a supplement to the analysis of the intact platform various damaged situations are simulated by removing X-braces. The damages are considered to be caused by dropped objects, ship collision or fatigue.

The collapse analyses are performed by monotonically increasing the environmental load until ultimate capacity. The calculations show that the jacket possesses a tremendous reserve strength in intact condition. The load at ultimate collapse ranges from 4.1 to 4.6 times the design load. Considerable redistribution of forces takes place beyond first yield hinge which occurs at a load of about 60-65 percent of the ultimate strength. This indicates that assessment of reserve strength with linear theory may be very conservative.

The most severe reduction in load-carrying capacity in damaged condition is associated with loss of the lower X-bracing in one of the end rows. In this case the ultimate capacity is 72 percent of the load in intact condition.

Example 6: Module support frame

The present study concerns a deck frame supported by four concrete shafts. The deck consists of two longitudinal main frames connected by transverse frames. The elements are welded rectangular box sections.

The deck is loaded by dead weight and operational loads on both levels. These loads are proportionally incremented in 26 steps up to collapse. The successive failure of elements was easily simulated. The deformed configuration of the frame is traced in Figure 13 while Figure 14 gives the load-deflection relation at midspan of a main girder.

Example 7: Residual strength of tripod

The final example concerns residual strength studies of the Hylight platform concept shown in Figure 15. One of the inclined braced elements is checked for progressive collapse after possible damages from dropped objects.

The axial load in the inclined element from global platform behaviour is increased until collapse. Figure 16 shows one analysis case indicating the bracing with damage. The collapse load for the damaged structure comes out to be reduced by 15 percent from the intact condition.

CONCLUSIONS AND FURTHER WORK

The main idea behind the structural program USFOS has been presented. It appears that the technique represents an interesting alternative to the conventional nonlinear finite element method in the sense that modelling work is reduced and computer time saved.

The program USFOS is especially aimed at progressive collapse analysis of framed offshore structures. It is the intention to further develop the program to take in effects like

- refined modelling of damaged bracing
- crack criteria for tubular joints
- temperature dependent material characteristics for application to fire analysis
- interaction formulas for I-beams in deck including shear
- interaction formulas for concrete

so that a version is obtained specialized for code checks of accidental load situations.

Emphasis will also be paid to develop more efficient pre- and postprocessing tools.

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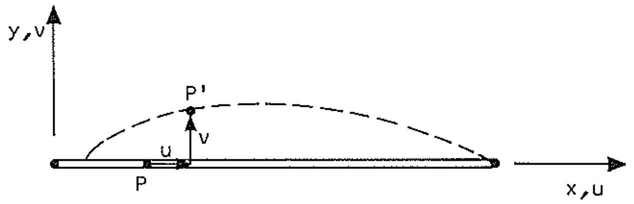


Fig 1 Local element xy–system

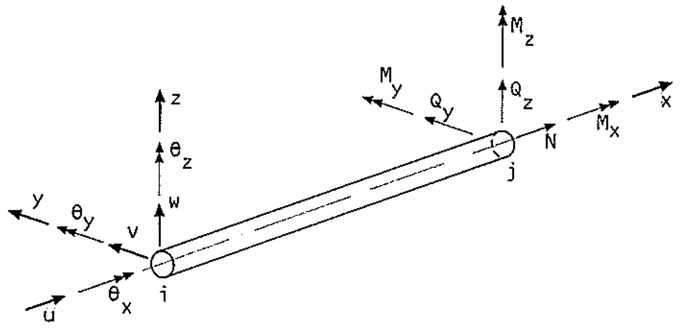


Fig 2 Three–dimensional beam element

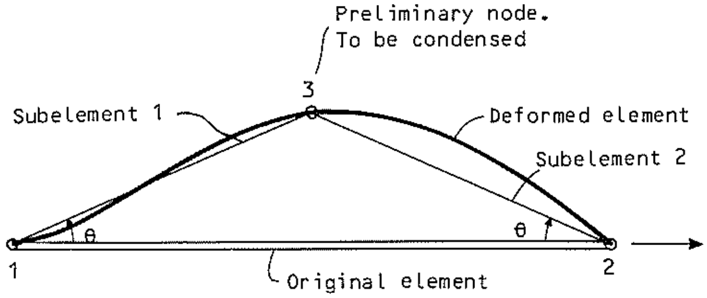


Fig 3 Subdivision of beam element for plastic hinge at midspan

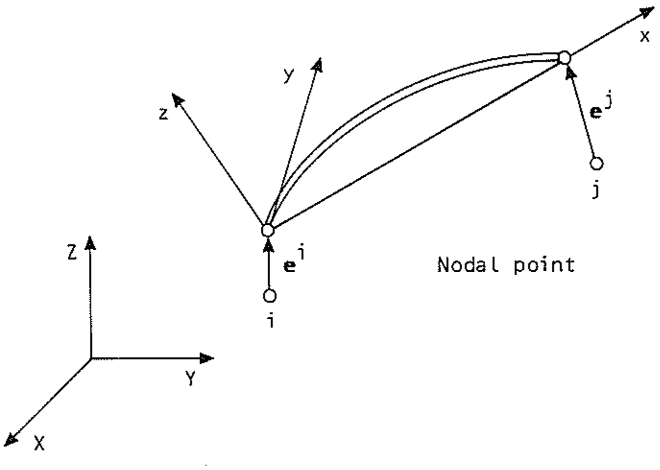
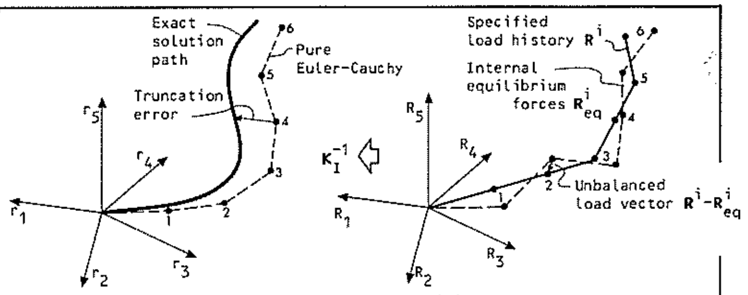


Fig 4 Beam element with node eccentricity in spatial system



(a) displacement space (b) load space

Fig 5 Displacement and load histories by Euler–Cauchy incrementation

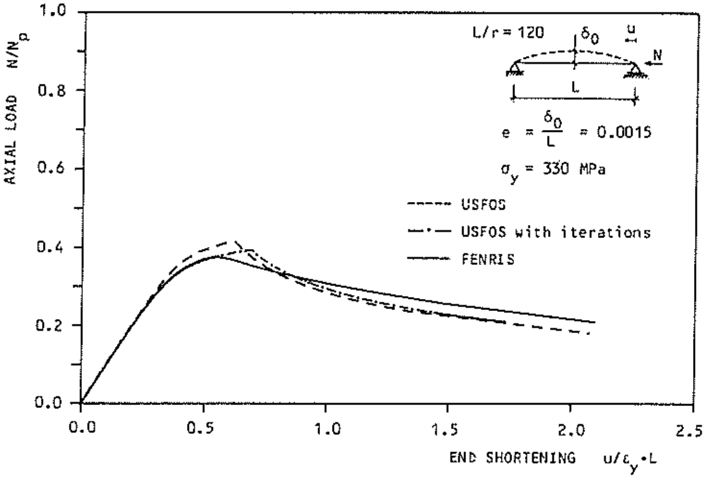


Fig 6 Load–end shortening curves for simply supported beam

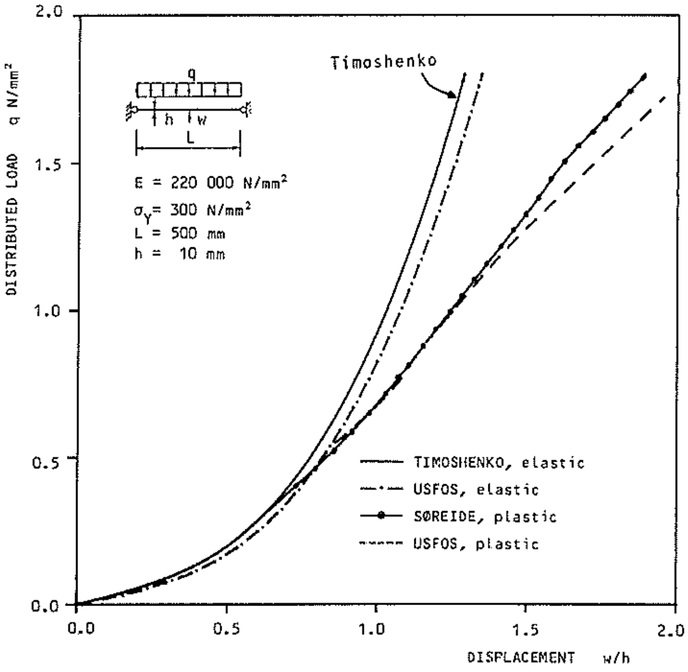


Fig 7 Load–deflection curves for simply supported beam

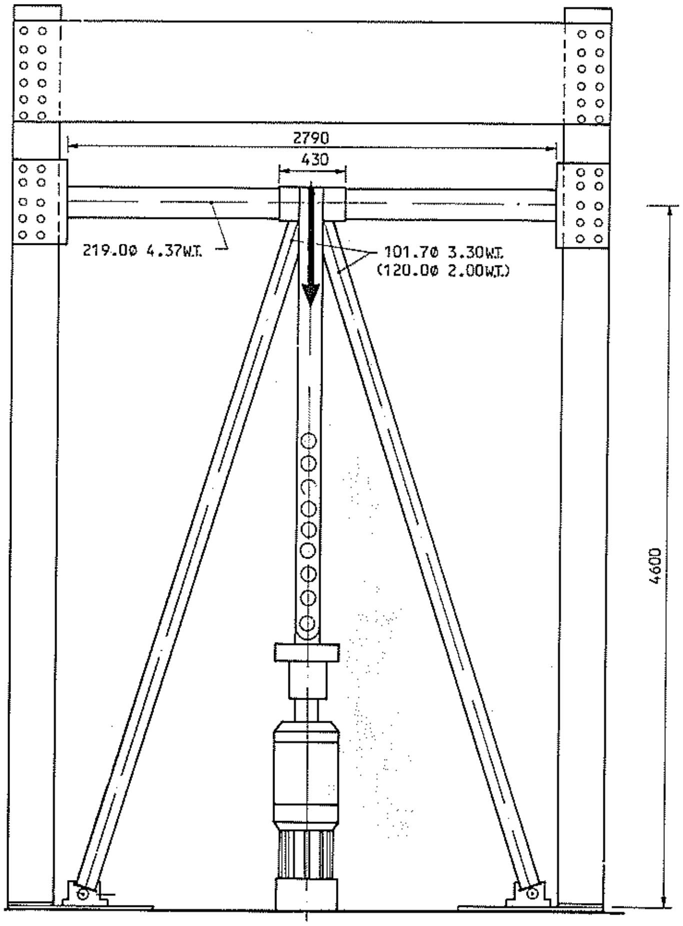


Fig 8 K–shaped frames

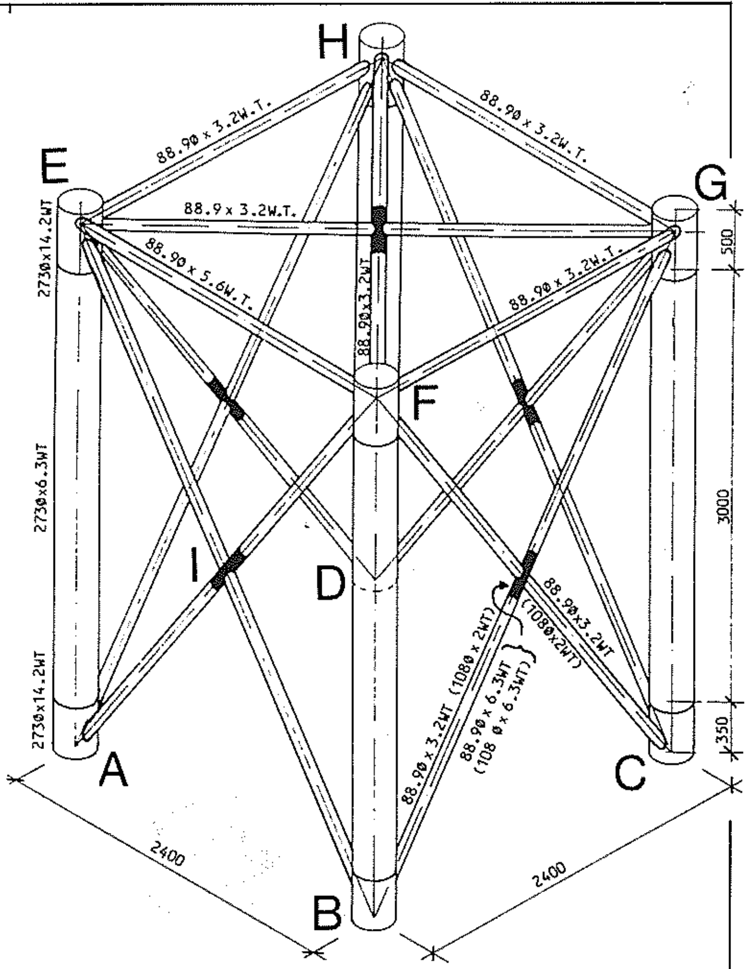


Fig 10 Member arrangement and dimensions for test frame S1 and S2. Figures in parenthesis refer to system S2.

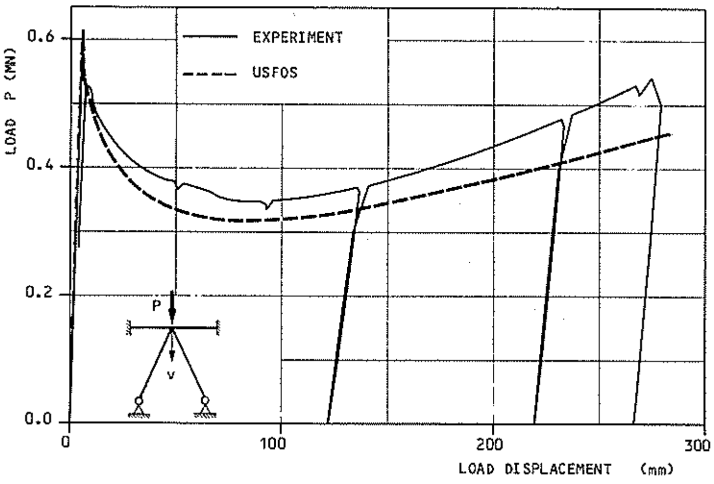


Fig 9 Load–displacement curves for K–frame

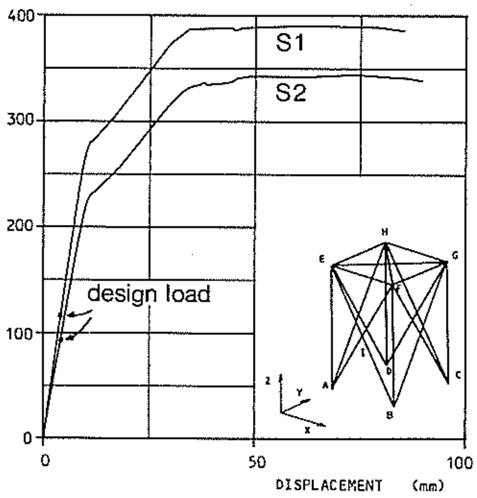


Fig 11 Load versus horizontal displacement in global x-direction of node E for test frames

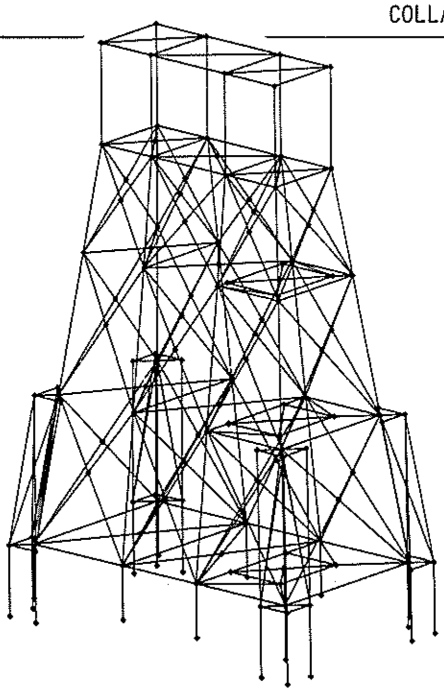


Fig 12 Eight–legged jacket

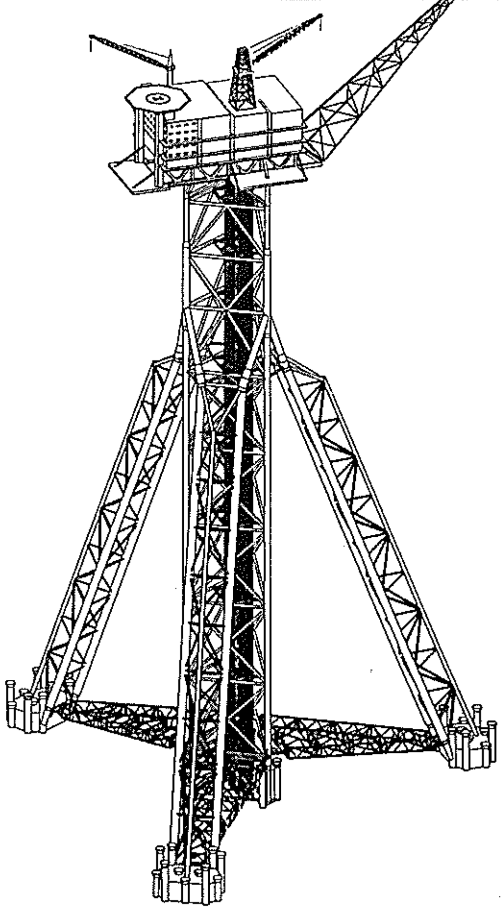


Fig 15 Hylight platform concept (Norsk Hydro)

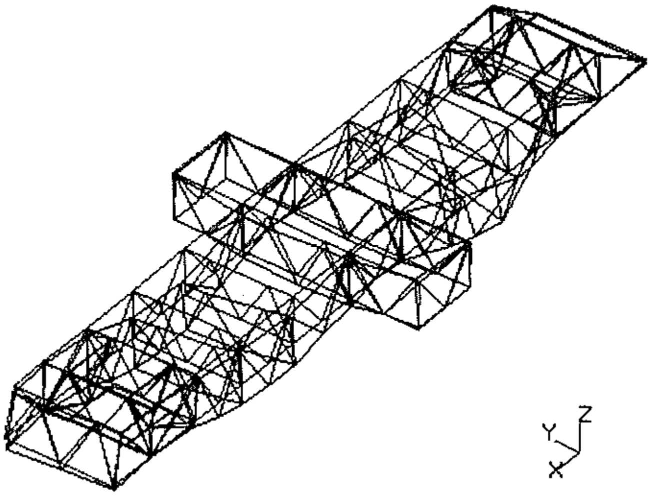
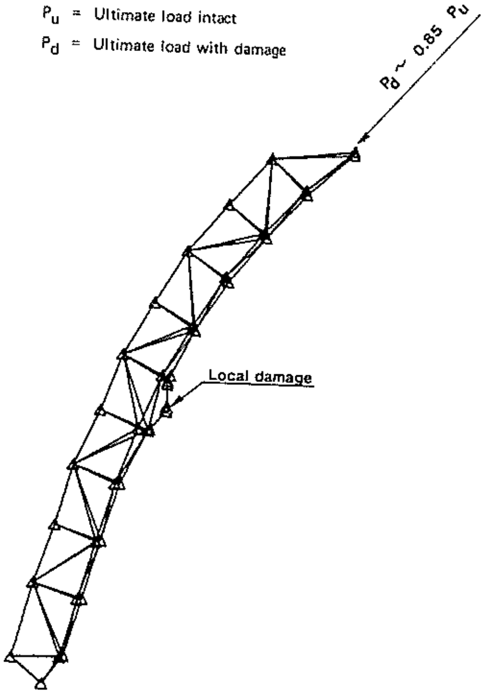


Fig 13 Deformed configuration of MSF



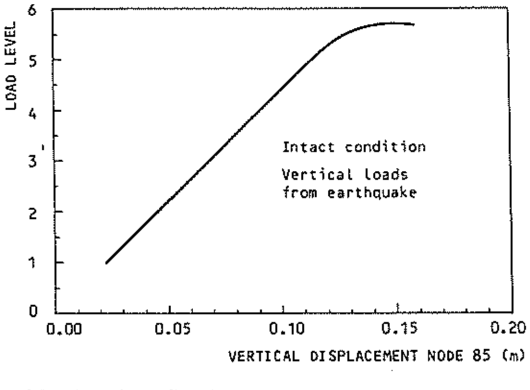


Fig 14 Load–deflection curve at midspan of main girder

Fig 16 Hylight platform with damage in inclined element. Configuration at collapse